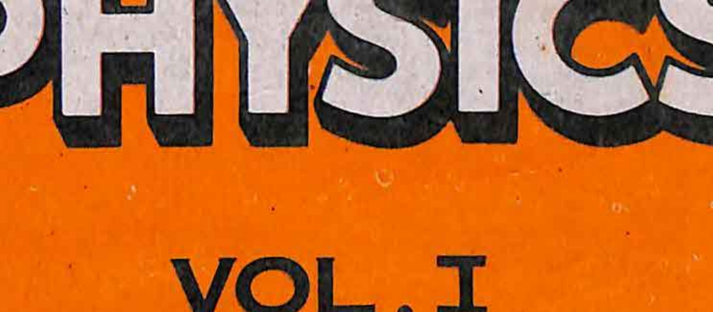
+2

Practical PHYSICS

VOL. I



K.K. MOHINDROO

PITAMBAR PUBLISHING COMPANY

+2

PRACTICAL PHYSICS

Volume I for Class XI

[STRICTLY ACCORDING TO THE NEW SYLLABUS PRESCRIBED BY THE CENTRAL BOARD OF SECONDARY EDUCATION, NEW DELHI, IN PRACTICAL PHYSICS FOR CLASS XI]

By
K. K. MOHINDROO
M.Sc., B.Ed., Dip. Ed. (U.K.)
Principal.
Air Force Bal Bharati School,
Lodi Estate, New Delhi.

S. C. E. R. T. West Bengal
Library
25/3, B. C. Road
Calcutta

PITAMBAR PUBLISHING COMPANY
Educational Publishers
888, EAST PARK ROAD, KAROL BAGH
NEW DELHI-110005 (INDIA)

© Reserved with the Author

First Edition : 1989
Second Edition : 1989
Reprinted : 1990

All rights reserved. No part of this book may be reproduced in any form without the written permission from the publishers.

Price : Rs. 25/-

Code : 27211

S.C.E.R.T. W.B. LIBRARY
Date: 31.5.06
Accn. No. 12194

Published by : PITAMBAR PUBLISHING COMPANY
888, East Park Road, Karol Bagh, New Delhi-110005.
Telepones : Office : 770067, 776058, 526933
Res. : 5715182, 5721321, 586788

Printed at PIYUSH PRINTERS PUBLISHERS (P) LTD.
G-12, Udyog Nagar, Rohtak Road Industrial Area, New Delhi-110041: Tel. 5472440

PREFACE TO THE FIRST EDITION

"Experiment is the interpreter of nature. Experiment never deceives. It is our judgement which sometimes deceives itself, because it expects results which experiment refuses."

—Leonardo da Vinci

We are living in a modern age. A complete and generous education is a prime necessity now, when the competition for existence is growing every moment and the struggle for international supremacy is threatening the very civilization.

We are moving fast, infact too fast. While the modern inventions have made our actions responsive to touch, the man of tomorrow must know how to keep pace with them. The future man is being manufactured in our class-rooms and laboratories now. To keep pace with his future environments, he must develop in him the technique of maximum output with the minimum of time, energy and material. The present day education ought to help him to acquire that. '*Plus Two Experimental Physics*' as a part of education, has a primary duty in this direction.

This book presents a text on experimental Physics intended to meet the needs of Class XI students opting for *new Physics courses* at the **Plus Two** stage of 10+2 year pattern of education. The book has been written after a long and direct contact of about 30 years with the students of Physics and in this period of contact I had the privilege to come across the actual problems confronting students in the laboratories.

The broad objectives of the new experimental course are :

(1) to develope curiosity of the students.

(2) to learn scientific method and develop creative ability through experiments and investigations, and

(3) to emphasise practical work in physics through interesting experiments, both short and long several of which can be done by using low cost items easily available.

The aim of the book is, however, not to replace the teacher. The teacher in the laboratory can always train a young pupil on proper lines ; the present book is to serve as an aid for this purpose.

The book is written in a simple and lucid language and is strictly according to the new syllabus prescribed by the Central Board of Secondary Education, New Delhi. **In the book, greater emphasis has been laid on the experimental, rather than the theoretical aspect of each experiment**. Of course, the theory and the principle of each experiment essential for its successful performance has been fully described and discussed. While describing the steps of procedure for the experiment logically and systematically, attention has been pointedly drawn to the important precautions to be observed at this stage. The record of observations is an important part of the procedure of the experiment. For this purpose simple and self explanatory tables have been drawn and, by way of illustration, actual readings have been inserted in some experiments and steps of calculation have been methodically shown. Whatever and whenever possible, graphical methods have been introduced.

Figures of instruments and diagrams required for the experiments have been neatly and accurately drawn. They are mostly sectional, simple and their important components have been labelled. Various vital points for drawing scientifically correct conclusions from the observations of an experiment have been discussed in sufficient details in the "INTRODUCTION".

A special feature of this book is that at the end of every experiment there are specimen **Oral Questions with their Answers** (which are usually asked at the time of Board Practical Examinations) concerning the experiment whereby the student is encouraged to test his grasp of the principles. This has been done with a purpose to lead the student to a better understanding of the experiment as a whole and thus to face the Practical Examinations with confidence.

I am thankful to several of my colleagues and friends and former students for their suggestions, criticism and help in the preparation of the book. I shall feel greatly obliged to those readers who will bring to my notice the shortcomings of the book and forward to me their suggestions for its improvement.

The book fully meets the requirements of the changed syllabus in Practical Physics for Class XI of Senior School Certificate Examination of the Central Board of Secondary Education, New Delhi.

The author wishes to express his special appreciation to the publishers, M/S PITAMBAR PUBLISHING COMPANY for their good work and painstaking efforts in bringing out this book in a short period of time.

PREFACE TO THE SECOND EDITION

In this edition, the experiments have been rearranged according to the various sections as per the new syllabus. I hope the book in its present form will prove more useful.

K.K.M.

NOTE

Students are required to perform **16 experiments in all,** at least **two experiments each from section A, B, C, D, E, F, G and H, in Class XI.**

Students will be required to conduct two experiments in the examination in Class XI.

Distribution of marks is as follows :

Two experiments	=10 × 2 = 20 marks.
VIVA	= 5 marks.
Records	= 5 marks.
Total	= 30 marks

CONTENTS

INTRODUCTION

(Accuracy, Errors, Graphs and Units) *(xiii—xxxi)*

SECTION A

Experiment - 1 1

(a) To find the least count of a vernier callipers.

(b) To find the internal and external diameters and depth of a cylindrical body (say a calorimeter) with the help of a vernier callipers.

(c) To calculate the internal volume of a cylindrical body (say a calorimeter) and confirm the same using a graduated cylinder.

Experiment - 2 10

To find the volume of the given metallic rectangular block by measuring its dimensions with a vernier callipers and check up the result by using a graduated cylinder.

Experiment - 3 12

(a) To find the least count of a screw gauge.

(b) To measure the diameter of a given wire by using screw gauge and compare it by wrapping the same wire around a pencil and explain the difference.

(c) To find the volume of the given wire by using screw gauge.

(d) To find the gauge of the wire using standard table.

(e) To find the thickness of a given glass strip using screw gauge.

Experiment - 4 18

To determine the inertial mass of a body.

Experiment - 5 23

To find the density of a substance of the given rectangular block using a beam balance.

Experiment - 6 31

To find the least count of the given stop-clock/watch and use it for checking the time period of oscillations of varying amplitude of a simple pendulum.

SECTION B

Experiment - 7 35

To study the rate of flow of water filled in a burette (or a vertical cylindrical container) connected to a capillary and to find its half-life.

Or

To find the time required to empty a burette full of water to $\frac{1}{2}$ of its volume; $\frac{1}{4}$ of its volume; $\frac{1}{8}$ of its volume and so on and to draw a graph between

(i) volume of water emptied (V) and time (t)

(ii) volume of water left and time and to study if at every stage the rate of flow is the same.

(iii) log V and t. What is the difference between these graphs? Explain.

Experiment - 8 41

To draw the s-t graph and v-t graph of the motion of hand as one walks using ticker-timer.

Experiment - 9 47

To study the changes in velocity of a body pulled by a constant force and measure its acceleration.

Experiment - 10 51

(a) To locate the position of no friction for the junction of three threads when it is in equilibrium under the action of three forces in the parallelogram law of vectors apparatus.

(b) To find the weight of the given body using the parallelogram law of vectors.

SECTION C

Experiment - 11 57

To find the downward force along the inclined plane acting on a trolley/roller on account of gravitational pull of earth and study its relationship with the angle of inclination of the inclined plane.

Experiment - 12 62

To study how the acceleration of a body depends on the force acting on it and the mass of the body.

Experiment - 13 70

To study the first law of motion of Galileo's experiment, using the double inclined track (i.e., a pair of inclined tracks each of which can be independently set at a small angle of inclination).

Experiment - 14 75

(a) To find the force of dynamic (sliding) friction for a wooden block on a horizontal plane.

(b) To find the force of rolling friction for a wooden block moving on rollers on a horizontal plane.

Experiment - 15 82

To study the relationship between force of dynamic friction and normal reaction, between a block and a horizontal surface by drawing a graph. Can this be identified with some physical quantity?

SECTION D

Experiment - 16 85

(a) To find the time-period of a simple pendulum and hence find the value of g i.e., acceleration due to gravity at a place.

(b) Draw a graph between l v/s t and l v/s t^2 and study which is better where l is the length of simple pendulum and t is the time period.
(c) To determine the length of the second's pendulum from the graph l v/s t^2.

Experiment - 17 95
To study the conservation of energy of a simple pendulum using ticker timer and a tape.

Experiment -18 99
To study the conservation of energy of a body falling freely using ticker time and tape.

Experiment - 19 102
To study the conservation of energy of a ball rolling down an inclined place (using a double inclined track).

Experiment - 20 103
To study the variation in the range of a jet of water by varying the angle of projection.

Experiment - 21 107
To plot a graph between the distance of the knife edges from the centre of gravity and the time period of a bar pendulum. From the graph find:
(a) the acceleration due to gravity at* ....
(b) the radius of gyration of the bar about an axis passing through the centre of gravity and perpendicular to its plane.

Experiment - 22 118
To find acceleration due to gravity with the help of a ball rolling down an inclined plane and by applying the correction for rotatory motion.

Experiment - 23 122
To find the acceleration of a freely falling body using a ticker time and a tape.

SECTION E

Experiment - 24 125
To study a Fortin's barometer and to measure the atmospheric pressure at a room temperature at a place.

Experiment - 25 130

To measure atmospheric pressure and study the variation of volume of a sample of air with its pressure at constant temperature. Also use the study to find the atmospheric pressure.

Or

To verify Boyle's Law for a gas and use it to find the atmospheric pressure.

Experiment - 26 138

To measure the air pressure inside a balloon by using an openended water manometer.

SECTION F

Experiment - 27 142

(a) To estimate the size of a molecule of an oil.

(b) To estimate the number of oil molecules in 1 gram-molecule.

Experiment - 28 147

To measure surface tension of water by capillary rise method.

Experiment - 29 159

To study the relationship between pressure and temperature of a sample of air at constant volume and hence determine the coefficient of increase of pressure of air at constant volume by Jolly's constant volume air thermometer.

Experiment - 30 166

To determine the comparative cleansing effect of the different detergents through the study of surface tension using capillary tube method.

Experiment - 31 168

To determine the surface tension of a liquid (water) by the dropweight method.

SECTION G

Experiment - 32 171

To determine the spring constant of a given spring.

Experiment - 33 174

To study graphically the stress — strain relationship for a steel wire using Searl's Apparatus and to determine the Young's Modulus for it.

Experiment - 34 184

To study the variation in length of a rubber band with applied force (beyond elastic limit).

Experiment - 35 189

To find the spring constant of a helical spring by measuring time period of vertical oscillations of a known load and check by measuring its extension by a known force.

SECTION H

Experiment - 36 194

(a) To study the relation between length and tension for constant frequency of a stretched string using a sonometer.

(b) Determine the frequency of a tuning fork.

Experiment - 37 205

To study the relation between frequency and length of a stretched string using a sonometer.

Experiment - 38 206

To study the relation between length and diameter for same frequency and tension of a stretched string using a sonometer.

Experiment - 39 209

To determine the frequency of A. C. mains using a sonometer. Taking the frequency of A. C. mains as 50 Hz, calculate the percentage error.

Experiment - 40 216

To find the difference in the frequencies of two tuning forks by counting their beats and identifying one of higher frequency.

Experiment - 41 220

To determine the wavelength of sound by taking first and second resonance positions and explain the difference, if any and hence determine velocity of sound at room temperature and the end correction. Also find the relation between frequency and Wavelength.

Experiment - 42 229
To compare the frequencies of two tuning forks using the resonance tube and find the end correction.

APPENDIX

Some Useful Constants and Tables
Table 1 —Densities of some common substances in kg m^{-3} 231
Table 2 —Acceleration due to gravity at different places in India along with their latitude, longitude and elevation. 231
Table 3 — Variation of atmospheric temperature and pressure with altitude. 232
Table 4 — Surface tension of liquids. 233
Table 5 — Elastic properties of solids. 233
Table 6 — Velocity of sound. 234
Table 7 — Standard wire gauge. 234
Table 8 — Coeffient of friction of common things. 235
Table 9 — Coefficient of expansion. 235
Table 10 — Density of water at various temperatures (in gm/c.c.) 236
Table 11 — Boiling points of water at various barometric heights. 236
Table 12 — Vapour pressure of water at various temperatures. 237

Logarithm Tables (i) to (xvi)

INTRODUCTION

(Accuracy, Errors, Graphs and Units)

"Nature is ever making signs to us; she is ever whispering to us the beginning of her secrets; the scientific man must be ever on the watch, ready at once to lay hold of nature's hint, however small; to listen to her whisper, however low." —*M. Foster*

In the world around us various natural phenomena are constantly occurring. The sun rises and sets causing the succession of day and night. The seasons follow in regular cycles. The weather undergoes constant changes. The stars, lightning, rainbows and many other things we notice have been the cause of wonder and curiosity to man.

Gifted with an inborn intelligence, man has constantly tried to know the cause of the natural phenomena and the natural changes, which he observes in his life. This enquiry carried on through generations, has led to a considerable amount of knowledge which we call **Science.** It is a Latin word which means 'to know'. In a general way, therefore knowledge of all kind is included in science, but in the ordinary accepted sense, *Science is that branch of knowledge which studies nature, its working and its laws.* Knowledge connected with non-living bodies or inert masses which go to make up the world is called the **Physical Science,** but *physics concerns itself with an investigation of matter and energy.*

Experiments. The growth of science has been very rapid in the last three hundred years. In this period, the experimental study of nature has been persued and it has proved extremely fruitful in revealing new knowledge, and as a means of deciding different theories, Galileo (1564-1642) may be called the father of experimental sciences. A process of taking observations under conditions deliberately arranged so as to answer a particular question is called **an experiment.** The object of performing an experiment of Physics in the laboratory is generally either to determine the value of some constant *e.g.*, acceleration due to gravity, value of earth's horizontal field, or to verify some principle, *e.g.*, Boyle's law, laws of refraction. An experiment affords the opportunity to a student to familiarise himself with various instruments about which he reads in theory. It further provides him with a golden chance to learn the habit of systematic observations and methodical procedure and thus train him in *"How to do things honestly, efficiently and regularly."*

Intelligent work is more important than good results; students who try to change their observations simply for getting accurate results, do not work with the true spirit of a scientist.

Accuracy of Observations

The fact that the accuracy of the final result is determined by the accuracy of the individual measurements and that all the measurements should be taken with upmost care is true but sometimes we may err on the other side, *i e.*, we may waste time in taking some reading too accurately. It is no use taking one observation to a much higher degree of accuracy than the other observations because it will not make the result more accurate. The accuracy of the result is the same as that of the least accurate observation.

The strength of a chain is known by its weakest link and so a good observer will always look at the weakest spot in his measurements. He would take the quantities one by one as they occur in the formula and would calculate the % error involved in each measurement with the instruments provided. Those quantities which involve large % error would need his keen attention and he would use, if possible, an instrument of greater precision for their measurement, and thus bringing them to the level of the other measurements.

In an experiment all observations are not equally important and the student should have a good idea of their relative importance so that the accuracy in a particular observation is not unnecessary pushed up. More attention should be paid to most important observations.

For example, consider a wire of correct diameter 1·00 mm. If by mistake it is found as 1·02 mm, the error of observation is 2%. Its correct and observed area of cross-section (πr^2) is 0·786 mm^2 and 0·817 cm^2 respectively and the error in its measurement is 4%. If the radius is raised to 4th power, the error would become 8%. *The % error in the result is as many times greater as is the power to which the quantity is raised. So the quantity having the highest power should be measured with highest precision than the rest.*

Accuracy Ordinarily Expected in a Measuring Instrument

The accuracy of a single observation made with a measuring instrument depends upon the instrument as well as on the skill and care with which the observation is made. Assuming that the instrument is correct and that reasonable care has been taken in carrying out the measurement we may expect that the error of a single observation lies within the limits mentioned below :

Instrument	*Error*
Metre-stick (in mm)	1 mm
Vernier Callipers (Vernier Constant=0·1 mm)	0·1 mm
Screw Gauge (Least Count=0·01 mm)	0·01 mm
Spherometer (,, ,, ,, ,,)	0·005 mm

Instrument	*Error*
Physical Balance (Sensitivity 1 mg per small division)	2 mg
Physical Balance (method of oscillation)	1 mg
Thermometer (0·5°C *div.*)	0·2°C (Using eye-estimation)
Thermometer (0·1°C *div.*)	0·05°C (Using eye-estimation)
Stop watch ($\frac{1}{5}$ *s.*)	$\frac{2}{5}$ *s.*

The table is meant to give the student a *rough* idea of the limit of accuracy and *should not be taken exact.*

The observations should be recorded with the same degree of accuracy as the measuring instrument permits. For example, in the measurement of the temperature with a half degree thermometer it is foolish to put down the reading correct upto a second place of decimal say 26·69, because the least count of the thermometer is $\frac{1}{2}$° and at the most we can estimate upto 1/10th of a degree. Thus with such thermometers the observations of temperature should be recorded only upto the first decimal place.

It is commonly used to denote the reliability of the indication of a measuring instrument, when taking an observation, fraction of the smallest divisions may be estimated and recorded as far as possible. For instance, *a length found to be exact 9 cms should be recorded as 9·0 cm to indicate that nearest mm has been read. Similarly, a temperature reading found to be exactly 25° should be registered as 25·0° to indicate that 10ths have been read and so for the mass if it is 76·5 g, it should be noted as 76·500 g to indicate that correct upto a mg has been weighed.*

You may find that in spite of all your efforts, there may be some difference between your calculated result and the standard value from physical constant tables. Absolute error is the difference between calculated result and the standard value. Remember, it is not the absolute error, but the percentage error which is the ratio of the absolute error to the total magnitude of the quantity multiplied by 100 that determines the accuracy of a measurement. If, for example, a student gets the value of g as 960 instead of 980, he is terrified by this large difference of 20 which is the absolute error, but he feels satisfied when he gets 0·000013 as the coefficient of linear expansion of iron in place of 0·000011. The first result, however, is only $\frac{20\times100}{980}$ or 2% approximately low, whereas in the second there is an error of $\frac{\cdot00002\times10}{\cdot000011}=18\%$. *If you find that your* *result does not tally with the standard value, do not cook it by changing your observation or writing fictitious reading or by doing wrong calculations but try to find out the cause of errors and state it in the sources of error.*

ERRORS

The quantities, which the students generally measure, may be mass, length, time, volume, temperature, current, potential difference, degree etc. In all these cases observation finally reduces to the reading of a scale or noting the coincidence between two marks. As the personal judgement of the observer is employed in estimating the coincidence between two marks or in recording the position of a pointer which is between two marks of a scale, so an error is inevitable. Besides, slight changes in experimental condition (*e g.*, small changes of temperature, pressure, voltage, etc.) may occur during the experiment and lead to small uncertain errors.

1. **Random Errors**. The errors arising out of small changes in the experimental conditions and the personal judgement of the observer are known as random errors. When random errors are due to entirely to chance, and not due to any personal bias on the part of the observer, they are likely to be positive as well as negative. When an observation gives a higher value than the true one, the error in the observation is said to be positive. *Large random errors due to chance are less likely to occur than the small.* If a large number of observations of the same quantity is made, it is likely that most of them will have small errors, each one of which has as much chance of being positive as negative. *Hence the arithmetic mean of several readings is likely to be closer to the true value than any one of the individual readings.*

Values to be Rejected. Sometimes it happens that one of the observed values differs rather widely from the rest. Should it be included in taking the arithmetic mean ? If the relative difference, when calculated in % is very large, it should be certainly rejected. But unfortunately the difference is not always so pronounced as would justify declaring it 'too large and fit to be rejected'. No golden rule could be laid down to say whether a particular observed value should be rejected or retained in taking the mean. To beginners, the following process based on practice will suit.

"Find the difference between arithmetic mean and each individual reading. Add up these differences, disregarding the signs and take their mean which will represent the *mean error*."

Let the values of focal length in cm of a lens as obtained by a student be (*i*) 37·2, (*ii*) 37·9, (*iii*) 38·3, (*iv*) 38·1, (*v*) 37·8, (*vi*) 38·0. The arithmetic mean is 37·9. The difference between the mean and the individual values are respectively (*i*) −0·7, (*ii*) 0·0. (*iii*) +0·4, (*iv*) +0·2, (*v*) −0·1, (*vi*) +0·1. When these are added up disregarding their signs, value is 1·5 and the mean value is $1{\cdot}5 \div 6 = 0{\cdot}25$. This value 0·25 represents the mean error and the student may express his value as 37·9±0·25 cm.

2. **Systematic Errors.** If during an experiment a factor operates in such a way as to make the observed value always higher or lower than the true value, this type of error is said to be systematic *e.g.*, instrumental errors (zero error, bench error, etc.) radiation loss or gain in calorimetry.

In the case where the source of systematic error is known, action should be taken to remove it. Thus, the zero error of a reading instrument is checked and the proper correction is applied to the reading. Error due to radiation loss is allowed to occur and then corrected for.

In some cases, the source of systematic error may not be definitely known but its existence is apparent. In such cases, the experiment is repeated under different conditions. If variation of conditions does not change the result, one may set his mind at rest that there is no systematic error.

Arithmetic Mean. When a large number of observations are required, as in the measurement of the diameter of a wire, the readings should be taken all over the length and in various directions, even if most of them happen to be the same. They should all be recorded and their arithmetic mean be taken. For taking a really independent reading in the repetition of an experiment, either, if possible some other part of the scale should be used or the magnitude of the various quantities should be altered and the result be calculated in each case. The mean result must then be found. If in an experiment on simple pendulum l_1, l_2, l_3 be the three widely different lengths and t_1, t_2, t_3 the corresponding time periods, then the *mean is not taken of the three lengths and the three time periods, but* l/t^2 *is calculated for each observation and the mean is taken of the nearly constant results of* l/t^2. It would be unnecessary to calculate 'g' for each observation and then get the mean value of 'g', for, in each case l/t^2 is to be multiplied by a constant $4\pi^2$.

Sometimes due to an accidental error of observation, a particular result comes out to be exceptionally too high or too low. This should be rejected or its observations, if possible, should be checked again, for it will otherwise mar the whole result. Mean cannot be taken when the different results are very much different from one another. In such cases the experiment should be performed again until the differences are reduced to within the experimental error.

Combination of Errors

(*a*) **Addition and subtraction.** If two quantities are to be added to or subtracted from each other, the error in the result is not obtained by using the fractional errors. The two errors are added together for both addition and subtraction and the result is expressed as a fraction or percentage of either the sum or the different of the two quantities. Thus if the two measured quantities have values $x \pm \delta x$ and $y \pm \delta y$ the value of the sum will be quoted as $x+y \pm (\delta x+\delta y)$, and the value of the difference will be $x-y \pm (\delta x+\delta y)$.

(*b*) **Multiplication and Division**. If two quantities x and y are multiplied together, the fractional error in the product is the sum of the two fractional errors in the x and y. Let δx, δy be the errors in x and y. Then the error in the product will be in the range :

$$(x \pm \delta x)(y \pm \delta y) - xy = \pm y\delta x \pm x\delta y.$$

We neglect $\delta x \delta y$ as it is small compared with $y\delta x$, $x\delta y$ etc., then the largest possible error (positive or negative) in xy will be $\pm(y\delta x + x\delta y)$ and the fractional error in xy will be $\pm\left[\left(\frac{\delta x}{x}\right)+\left(\frac{\delta y}{y}\right)\right]$ *i.e.*, the sum of the fractional errors in x and y.

The largest value of the quotient will be $\frac{(x+\delta y)}{(y-\delta y)}$ *i.e.*, an error of $\left[\frac{(x+\delta x)}{(y-\delta y)}-\frac{x}{y}\right]$ or a fractional error of

$$\left[\left(\frac{x+\delta x}{y-\delta y}\right)-\frac{x}{y}\right] \div \frac{x}{y} = \frac{xy+y\delta x-xy+x\delta y}{y\,(y-\delta y)} \times \frac{y}{x} = \left[\frac{\delta x}{x}+\frac{\delta y}{y}\right],$$

neglecting δy in comparison with y.

The smallest value of the quotient will be $\frac{(x-\delta x)}{(y+\delta y)}$ and a similar calculation shows that the fractional error is now $-\left[\left(\frac{\delta x}{x}\right)+\left(\frac{\delta y}{y}\right)\right]$. Thus the quoted error in $\frac{x}{y}$ will be $\pm\left[\left(\frac{\delta x}{x}\right)+\left(\frac{\delta y}{y}\right)\right]$.

(*c*) **Powers**. It is obvious from the above that, if the same quantity x is multiplied together 'n' times (*i.e.*, x^n is obtained), the fractional error in x^n is 'n' times the fractional error in x. It is not quite so obvious that, if the nth root of x is taken, the fractional error is $\frac{1}{n}$ of the fractional error of x, so we shall prove it using the binomial theorem.

$$\text{Error} = (x+\delta x)^{1/n} - x^{1/n} = x^{1/n}\left(1+\frac{\delta x}{x}\right)^{1/n} - x^{1/n}$$

$$= x^{1/n} + x^{1/n}\frac{\delta x}{nx} - x^{1/n} = x^{1/n}\frac{\delta x}{nx}$$

$$\text{Fractional error} = \frac{1}{n} \cdot \frac{\delta x}{x}$$

(*d*) **Logarithmic Functions.** If the final result contains a logarithmic function, there are two ways of dealing with the error. In the first, the logarithms of the mean value plus and minus the error are found, the difference taken, and this is expressed as a fraction of the log of the mean value. Secondly,

$$y = \log_e x$$

$$\delta y = \frac{\delta x}{x},$$

$$\therefore \quad \frac{\delta y}{y} = \frac{\delta x}{xy}$$ and so the error can be calculated.

(*e*) **Trigonometrical Functions.** Quite often the final result depends upon the sine or some other trigonometrical function of an angle that has been measured. The fractional error wanted will be that of the function and not of the measured angle. Suppose an angle θ has been measured with an error of $\delta\theta$, and the final result includes the expression $\sin\theta$.

$$\text{error in } \sin\theta = \delta(\sin\theta) = \cos\theta\,\delta\theta.$$

$$\text{fractional error} = \frac{\delta\theta}{\tan\theta.}$$

Now $\tan\theta$ can have all values from 0 to ∞ so that the fractional, error can have all values from infinity to 0.

Suppose the critical angle in a medium is measured as $40^\circ \pm 1^\circ$. What is the fractional error in the value of the Refractive Index ?

$$\mu = \frac{1}{\sin\theta} = \operatorname{cosec}\theta$$

$$\delta\mu = -\cot\theta\,\operatorname{cosec}\theta\,\delta\theta$$

$$\frac{\delta\mu}{\mu} = -\cot\theta\,\delta\theta,$$

$$\cot 40^\circ = 1{\cdot}19\ ,\ \delta\theta = 1^\circ = \frac{\pi}{180} = \frac{1}{60}$$

$$\frac{\delta\mu}{\mu} = \frac{1{\cdot}19}{60} = \frac{1}{50}$$

$$\mu = \operatorname{cosec} 40^\circ = 1{\cdot}55 \text{ to 1 part in 50}$$

$$= 1{\cdot}55 \pm 0{\cdot}03$$

Suppose $y = \tan x$

$$\delta y = \sec^2 x\,\delta x$$

$$\frac{\delta y}{y} = \frac{1}{\cos x \sin x}\,\delta x = \frac{2\delta x}{\sin 2x}.$$

This is obviously smallest when $\sin 2x$ is greatest, *i.e.*, $x = 45^\circ$. If it is possible to choose, one chooses, angles close to 45°.

Examples on the Calculation of Errors

Example 1. The following observations were actually made during an experiment to find the value of 'g' using simple pendulum.

Length of simple pendulum $= l = 100$ cm

Time for 20 vibrations $= 40$s

Now $T = 2\pi \sqrt{\frac{l}{g}}$ when $T =$ time period

$l =$ length of the simple pendulum

$g =$ acceleration due to gravity

$\therefore \quad g = 4\pi^2 \frac{l}{T^2} = 4\pi^2 \frac{l}{\left(\frac{t}{20}\right)^2}$ where $t =$ time for 20 vibrations.

or $g = 4\pi^2 l . \frac{(20)^2}{t^2}$.

Taking log of both sides

$$\log g = \log 4 + 2 \log \pi + \log l + 2 \log 20 - 2 \log t$$

Differentiating, we have

$$\frac{\delta g}{g} = 0 + 0 + \frac{\delta l}{l} + 0 - \frac{2\delta t}{t}.$$

Converting the negative errors into positive ones for maximum effect, we have

$$\frac{\delta g}{g} = \frac{\delta l}{l} + \frac{2\delta t}{t}$$

$\therefore \frac{\delta g}{g} = \frac{0{\cdot}1}{100} + \frac{2 \times 0{\cdot}1}{40}$ as $\delta l = 0{\cdot}1$ cm $=$ least count of metre scale

and $\delta t = 0{\cdot}10 =$ least count of stop watch.

$\therefore \frac{\delta g}{g} = 0{\cdot}001 + 0{\cdot}0050 = 0{\cdot}0060.$

$\therefore$ Maximum possible error $= 0{\cdot}0060 \times 100$

$= 0{\cdot}60\%$.

As maximum contribution to total error is $0{\cdot}5\%$ (*i.e.*, $0{\cdot}005 \times 100$) so 't' should be measured very carefully.

Example 2. The following observations were actually made during an experiment to find the radius of curvature of a concave mirror using a spherometer.

$l = 4{\cdot}4$ cm, $h = 0{\cdot}085$ cm

Now $R=\frac{l^2}{6h}+\frac{h}{2}$.

Taking log of both sides

$$\log R=2\log l-\log 6-\log h+\log h-\log 2$$

Differentiating both sides

$$\frac{\delta R}{R}=\frac{2\delta l}{l}-0-\frac{\delta h}{h}+\frac{\delta h}{h}-0.$$

Converting the negative errors into positive ones for maximum effect, we have

$$\frac{\delta R}{R}=\frac{2\delta l}{l}+\frac{2\delta h}{h}$$

$=\frac{2\times 0{\cdot}1}{4{\cdot}4}+\frac{2\times 0{\cdot}001}{0{\cdot}085}$ as $\delta l=0{\cdot}1$ cm=Least count of metre scale.

and $\delta h=0{\cdot}001$ cm=Least count of spherometer.

$=0{\cdot}045+0{\cdot}023$

$=0{\cdot}068$

∴ Maximum possible error $=0{\cdot}068\times 100$

$=6{\cdot}8\%$.

It is important to note that the maximum contribution to total error in the result is due to the individual error *i.e.*, 4·5% in the measurement of distance (*l*) between the fixed legs of the spherometer. Hence '*l*' should be measured most carefully.

GRAPH

A *graph* is a line, straight or curved, showing the relation between two variable quantities (*or* their powers *or* functions) of which one varies as a result of the change in the other. It has, however, a much deeper significance.

Just as no amount of a wordy description of a landscape can convey the same, vivid idea to the mind as a painting or a nice photograph, so also no amount of data, however nicely recorded, can give as clearly an idea of the relationship between two variable quantities as a graph. A graph has the same utility to a scientist as a picture for a layman.

In hospitals, regular temperature charts are kept of all patients so that the doctor may at once see how the patient's temperature is rising or falling. This would not be possible if he was given figures of temperatures and time, however, nicely entered in a tabular form. Not only this, these days graphs are widely used in almost all departments. There are graphs showing the increase and decrease of revenue in various government departments. You can also have a graph how your income and expenditure varies monthly.

A graph gives not only the relation between two variable quantities, it also enable us :

1. To determine the value of a quantity not actually observed during the experiment, *e.g.*, the temperature of a patient can be known from the graph for a time when it was not actually taken.

2. To verify certain laws *e.g.*, Boyle's law, it tells us how far we have succeeded in taking our observations correctly, for if we do not obtain the required form of a graph for the given relation, we have obviously done error somewhere. The graph shows the magnitude of the error also.

3. To calibrate certain instruments and to determine their true readings, *e.g.*, ammeter, voltmeter, etc.

How to Plot a Graph. Graphs are generally plotted on a paper ruled in millimetre square. The following are few simple rules that will help a student in plotting a good graph.

1. For drawing a graph, take at least six observations extending over a wide range.

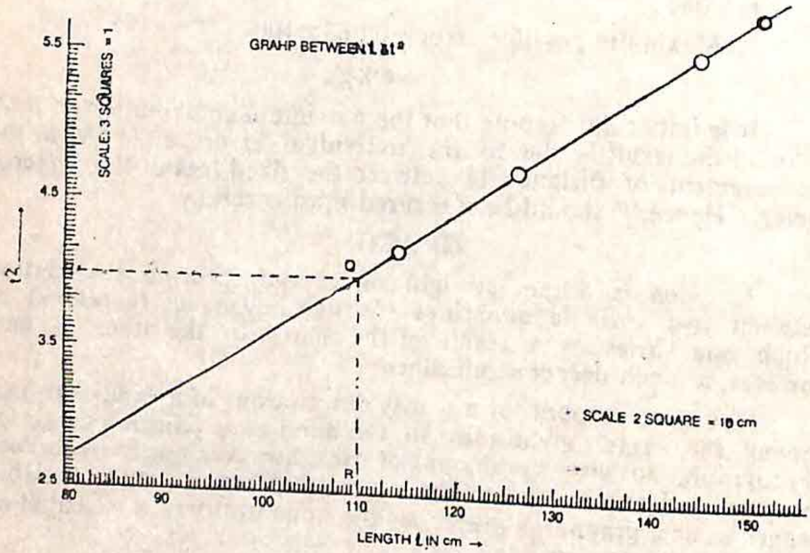


FIG (i)

Fig. (i)

2. In case all the observations are positive, draw two thick lines at right angles to each other at the bottom and at the left hand edge of the paper to indicate the axes of reference, the horizontal

line as the *abscissae* or *x-axis* and the vertical lines as the *ordinate* or *y-axis* intersecting at a point O, called the *origin*. If the observations also include some negative quantities, the origin may be taken somewhere in the middle of the paper.

3. As a general rule, the quantity that is made to alter at will is called the *independent variable* and is plotted against the x-axis and other which varies as a result of, this change is called *dependent variable* and is plotted against y-axis. Write clearly under the two axes the quantities which are represented by them.

Examples. (*a*) The relation between natural numbers (*independent variable*) and their squares (*dependent variable*) can be easily represented by a graph.

(*b*) In the simple pendulum where the time period is measured for different lengths by changing the length, *length* is the independent variable and the (*time period*)^2 or (t^2) the dependent variable.

(*c*) In the verification of Boyle's Law where the volume is measured for various values of pressure, *pressure* is independent variable and *volume* (or 1/V) the dependent variable.

Sometimes for the sake of convenience, the independent variable is represented along y-axis and the dependent variable along x-axis *e.g.*, in the case of simple pendulum it is more convenient to take 'l' along the y-axis and t or t^2 along the x-axis.

4. The scale along each axis should be chosen with a view to utilise the large part of the graph paper and at the same time retaining convenience of plotting. *The unit of length on the graph paper and the unit of the quantities to be plotted should bear, to each other, a simple ratio*. To achieve this, the round number, nearest and slightly less than the minimum value in the data should be taken near the origin and the nearest round number to the maximum value and slightly greater at the end of the required axis, *the successive value being laid along the axis in an ascending order*.

The student need not be disheartened if his points do not fall on the actual line or curve. They are the result of, and in a way a measure of, the amount of unavoidable experimental errors and his honesty.

When is a Straight Line Graph Expected? The graphs generally drawn in Physics should be straight lines, as the student may judge by mere look whether a graph is a straight line where

S.C.E.R.T. West Bengal
Library
Calcutta
25/3, B.C. Road

S.C.E.R.T., W.B. LIBRARY
Date..................
Acc. No.................

as it is difficult to say whether it forms the part of an ellipse or a parabola. The attempt, therefore, always is to plot graphs between two such quantities as will yield a straight line.

In order to understand when to expect a straight line graph, let us study the graph shown in Fig. (*ii*). Here the ratio $y/x = y_1/x_1$ or broadly speaking y/x is constant for every point on the graph and is equal tan θ, where θ is the angle that the graph makes with the x-axis. Thus, whenever the ratio y/x between two variable quantities is constant, one should confidently expect the graph to be a straight line.

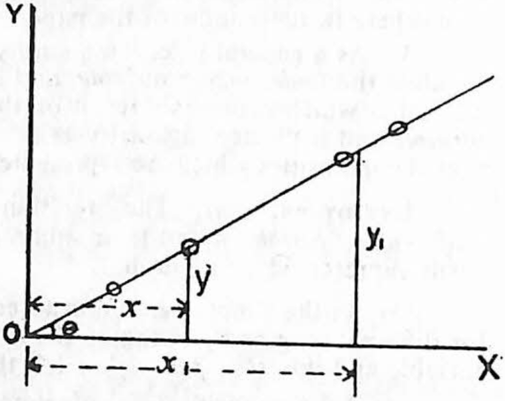


FIG (ii)

As in Boyle's Law PV or xy is constant and not P/V or x/y, the student should put his quantities in the form of x or P and $1/y$ or $1/V$ so that $x \times 1/y$ or $P \times 1/V$ will again be a constant and the resulting graph between P and $1/V$ is a straight line. It is for this very reason that a graph between length and (time period)^2 is generally plotted in the case of a simple pendulum.

The reader will probably already know that any equation relating two unknown quantities y and x in which no powers higher than the first occur will yield a straight line when y is plotted against x. We can arrange any such linear equation in the form $\boxed{y = mx + c}$. The quantity 'm' is called the gradient or slope of the line. Lines with the same value of 'm' are all parallel Fig. (*iii*). The quantity 'c' is a measure of the distance 'OB' and is called the intercept on the y-axis ; the intercept on the x-axis is $\frac{-c}{m}$. When a straight line graph is used as a source of information we nearly always measure the slope or intercepts or both.

Measurement of the Slope of Line. Select two points A, B on the line drawn—not necessarily observational points—which are well separated Fig. (*iv*). Record from the graph the values of y_A and x_A (at A) and of y_B and x_B (at B). Then the slope is

$$\frac{y_B - y_A}{x_B - x_A} = \frac{BC}{CA}.$$

It must be clearly understood that the distances BC and CA must be measured in the units of the scales of the graph and *not* in centimetres or in 'squares'. They are not merely distances *e.g.*, if OY represents volts and OX amperes, the slope would be measured in ohms.

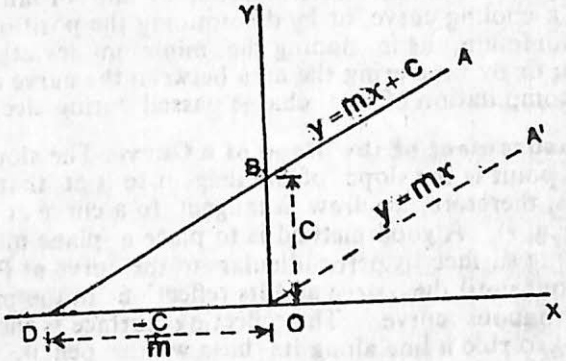


FIG (iii)

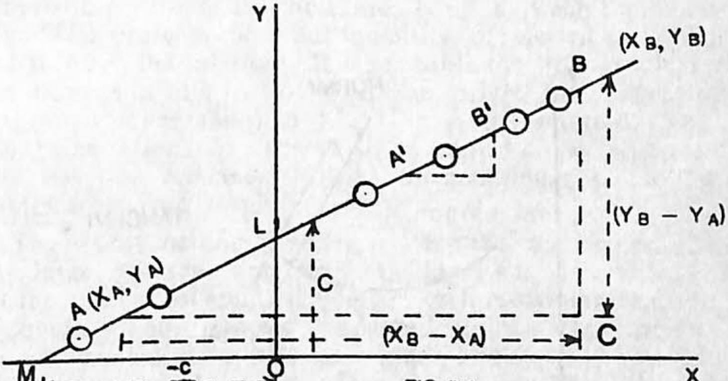


FIG (IV)

Measurement of Intercepts. If as in Fig. (*iv*), the origin O from which the two scales are measured appears on the paper, it is easy to read off the intercepts OL and OM directly. Sometimes, however, the scale, chosen for quite sound reasons is such that the origin does not appear on the paper. In such instances two well separated points are selected as before and the values of y_A, x_A, y_B, x_B are read from the scales as before. Then $y_A = mx_A + c$ and $y_B = mx_B + c$, so that by solving these equations the values of m, c and $\frac{-c}{m}$ may be found. Alternatively the intercept on one axis can often be read from the graph and then if m is measured, we can calculate the other.

Deductions from curved graphs. Quantitative information is usually derived from curves by measuring the slope of the curve at particular points as in the determination of rate of fall at temperature from a cooling curve, or by determining the position of a maximum or minimum, as in finding the minimum deviation produced by a prism or by measuring the area between the curve and the axis, as in the computation of total charge passed during electrolysis.

Measurement of the Slope of a Curve. The slope of a curve at a given point is the slope of the tangent to it at that point. The problem is, therefore, to draw a tangent to a curve at a particular point P (Fig. *v*). A good method is to place a plane mirror so that the reflecting surface is perpendicular to the curve at P. Turn the mirror about until the curve and its reflection in the mirror appear as one continuous curve. The reflecting surface is then normal to the curve ; so rule a line along its base with a pencil. The tangent is then the line through P at right angles to this normal and can be drawn with a set square and its slope can be measured as for any other straight line.

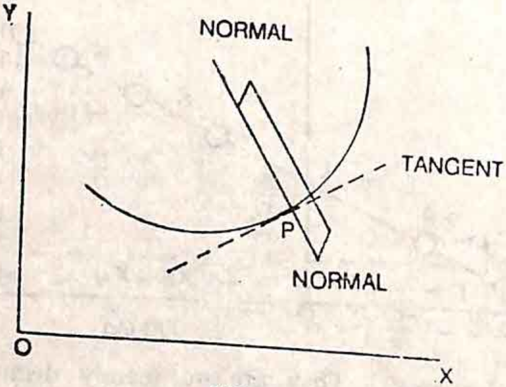


FIG (v)

Measurement of the Area Under a Curve. It is sometimes necessary to measure the area lying between a curve—or a straight line—and one of the axes. For example, if the current I during an electrolysis experiment is read at regular intervals during its performance and then I is plotted against *t*, the time, a curve of the type shown in Fig. [*vi* (*a*)] may be obtained. The area under the curve *i.e.*, the area lying between the curve and the time axis, will represent the total charge passed in coulombs. For during any small interval '*t*' the current may be considered to have a constant value I and so

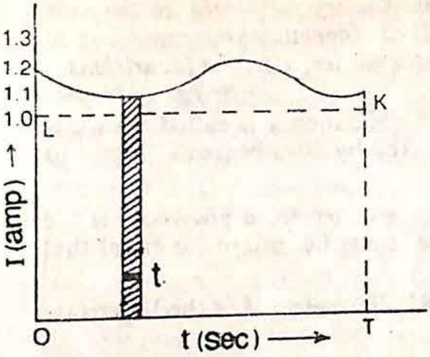


FIG [vi (a)]

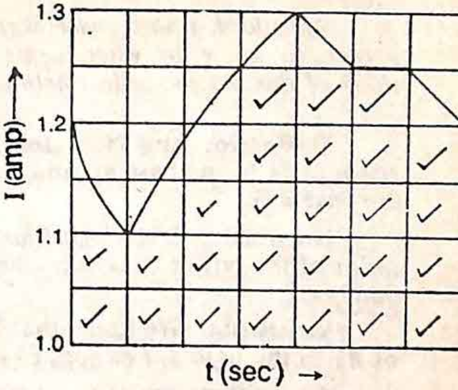


FIG [vi (b)]

the area shaded in Fig. [*vi* (*a*)] represents the charge passed during this short-time. Hence the total area lying between the curve and the line OT represents the total quantity of electricity which has passed through the solution. It is probable that the graph may be drawn more as in Fig. [*vi* (*b*)] with true origin not appearing *i.e.*, with the area under the line LK [Fig *vi* (*a*)] omitted. This will enable the variations in current to be shown on a large-scale. If such is the case remember to add the area under LK to the total computed from your graph.

The easiest method for determining the area is first to count all the large squares in the area, ticking them as recorded. Then count the number of small squares in each remaining portion of a large square. Ignore fractions of small squares less than half and count all greater than half as complete squares. Then add any correction for area not shown. Finally work out from the scales of that two axes what each square represents and multiply. Thus in Fig. [*vi* (*b*)] if one large square vertically represents 0·1 amp. and one large square horizontally represents 30 s. the area of each large square will represent $0{\cdot}1 \times 30 = 3$ coulombs.

In all cases it is important to check that the area between the curve and the correct axis measured.

LOGARITHMS

[Logarithm which literally means "a rule to shorten Arithmatic", "reduces to few days the labour of many months, and so doubles, as it were, the life of a mathematician besides freeing him from the errors and digest inseparable from long calculations."]

—*Laplace*

Logarithms were invented by Napier.

"My lord, I have undertaken this journey purposely to see your person, to know by what engine of wit or ingenuity you came first to think of this most excellent help in Mathematics, viz., the logarithms.

—Briggs to Napier

Definition and Notation. If $a^x = N$; then x is called the *logarithm* of N to the base a, and is denoted by $\log_a N$ (read "log N to the base a").

[In words : The *logarithm of a number to a given base* is the *index* of the power to which the base must be raised to equal that number.]

Example. We know that $3^4 = 81$. Therefore 4 is the logarithm of 81 to the base 3, *i.e.*, $\log_3 81 = 4$.

Logarithms are of two types : (*i*) *Naperian* in which the base is e. They are used in practice. (ii) *Common*, in which the base is 10. They are commonly used. When the base is not mentioned, it is implied that the base is 10. Four-figure Logarithm Tables of natural numbers to the base 10 are given at the end of this book.

Common Logarithms

Logarithm of a number consists of two parts :

(1) *Characteristic* is the integral part (whole or natural number).

(2) *Mantissa* is the fractional part generally expressed in decimal form.

Note : The *mantissa* is always positive.

How to find the characteristic of a number. The characteristic depends on the magnitude of the number and is determined by the position of the decimal point. *For number greater than one, the characteristic is positive and one less than the number of digits to the left of the decimal point. For numbers smaller than one (decimal fraction), it is negative and one more than the number of zeros between the decimal part and first digit.*

Examples : Characteristic of

530800 is 5
5308 is 3
53·38 is 1
5·308 is 0
·5308 is —1
·05308 is —2
·0005308 is —4
·00005308 is —5

The negative characteristic is usually written as $\bar{1}, \bar{2}, \bar{4}, \bar{5}$, etc., and is read as bar 1, bar 2, etc.

How to find the Mantissa of a number. The value of mantissa depends on the *digits* and their *order*, and is independent of the position of the decimal point. As long as the digits and their order is the same the mantissa is the same, whatever be the position of the decimal point.

The logarithm tables give the mantissa only. They are usually meant for number containing four digits, and if a number consists of more than four digits, it is rounded off to four digits 632562 should be taken as 6326 and 632537 as 6325. To find mantissa tables are consulted in the following manner :

1. The first two significant figures of the number are found at the extreme left vertical column of the table wherein the number lying between 10 and 99 are given. The mantissa of the digits which are less than 10 can be determined by multiplying the figures by 10, *i.e.*, the mantissae of 2, 20, 200 and so on, is the same.

2. Along the horizontal line in the topmost column the figures | 0 | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 10 | are given. They correspond to the third significant figure of the given number.

3. The difference for the fourth significant digit is given further in difference column under | 1 2 3 | 4 5 6 | 7 8 9 |. From 10 to 19 double differences are given. If the third figure is from zero to four the upper line should be read and for the figures from 5 to 9, lower line.

Example 1. *Find the logarithm of 368·6*.

The number has 3 digits to the left of the decimal point. Hence its characteristic is 2.

To find the mantissa ignore the decimal point and look for 36 in the first vertical column and 8 in the central topmost column. Proceed from 36 along a horizontal line towards the right and from 8 vertically downwards. The two lines meet at a point where the number 5658 is written. This is the mantissa of 368. Proceed further along the horizontal line and look vertically below the figure 6 in the differences column. You will find the figure 7 there. Hence the mantissa of 3686 is 5658+7=·5665.

∴ The logarithm of 398·5 is 2·5665.

Example 2. *Find the logarithm of 368600*.

The characteristic of this number is 5 and the mantissa is the same as in Example 1.

∴ Logarithm of 368600 is 5·5665.

Example 3. *Find the logarithm of ·00368633*.

The characteristic of this number is $\bar{3}$ as there are two zeros following the decimal point. We can find the mantissa of only 4 significant digits. Hence we neglect the last 2 digits (33) and find the mantissa of 3686 which is ·5665.

∴ Logarithm of ·0036833 is $\bar{3}$·5665.

When the last figure of a number consisting of more than 4 significant digits is equal to or more than 5 the digit next to it is raised by one and so on till we have only 4 significant digit and if the last digit is less than 5 it is neglected as in the last example.

Imp. If we have the number 368658, the last digit is 8, therefore, we shall raise the next digit to 6 and since 6 is again more than 5, we shall raise the next digit to 7 and find the logarithm of 3687.

Antilogarithm. The number whose logarithm is x is called the *antilogarithm* of x, and is denoted by *antilog* x.

Thus since log 2=·3010, antilog ·3010=2.

Example 1. *Find the number whose logarithm is 1·6078.*

(*i*) The characteristic=1. This is one less than the number of digits in the integral part of the required number, hence the number of digits in the integral part of the required number is 1+1=2.

(*ii*) Removing the integral part 1 from the given logarithm we get ·6078. The *first two digits* from the left are 60, the *third digit is* 7 and the *fourth* is 8.

(*iii*) In the table of the antilogarithms, look in the first vertical column for ·60. In this horizontal row under the column headed by 7, we find the number 4046 at the intersection.

(*iv*) In the continuation of this horizontal row and under the mean differences column on the right headed by 8, we find the number 7 at the intersection. Adding 7 to 4046, we get 4053.

(*v*) Antilog 1·6078=2+·4053=2·4053.

Example 2. *Find the antilogarithm of* $\bar{2}$*·06078.*

As the characteristic is $\bar{2}$, there should be one zero on the right of decimal in the number. Hence antilog $\bar{2}$·6078=·04053.

Laws of Logarithms. From the laws of indices we obtain the following laws of logarithms :

1. $\log_a mn = \log_a m + \log_a n$
2. $\log_a m/n = \log_a m - \log_a n$
3. $\log_a m^n = n \log_a m$

Two Important Special Cases

(*i*) $\log_a 1 = 0$ [The logarithm of 1 to any base is zero] as $a^0 = 1$.

(*ii*) $\log_a a = 1$ [The logarithms of the base to itself is 1] as $a^1 = a$.

Example 1. *Volume of a cylinder of length 3·47 cm is 1·932* cm^3*. Determine its radius.*

Volume $(V)=\pi r^2 l$

$$r^2=\frac{V}{\pi l}$$

$$r=\sqrt{\frac{V}{\pi l}}=\sqrt{\frac{1{\cdot}932}{3{\cdot}142\times 3{\cdot}47}}$$

$$\log r=\tfrac{1}{2}\,(\log 1{\cdot}932-\log 3{\cdot}142-\log 3{\cdot}47)$$

$$=\tfrac{1}{2}(0{\cdot}2860-0{\cdot}4972-0{\cdot}5403)$$

$$=-\tfrac{1}{2}\times{\cdot}7515=-{\cdot}3758=\bar{1}{\cdot}6242$$

[**Note.** The mantissa should always be in *positive* even though the logarithm comes out in negative.]

Taking antilogarithm of $\bar{1}{\cdot}6242$

We get $r={\cdot}4209$ cm

Basic S.I. Units

1. Length—metre (m) 2. Mass—kilogram (kg) 3. Time—second (s) 4. Temperature—kelvin (K) 5. Amount of Substance—mole (mol) 6. Intensity of illumination—candela (cd) 7. Current—ampere (A).

Derived Units

1. Energy—joule (J) 2. Force—newton (N) 3. Area—square metre (m^2) 4. Volume—cubic metre (m^3) 5. Density—kg/m^3 6. Velocity—metre per second (m/s). 7. Acceleration—metre per sec per sec (m/s^2). 8. Frequency—hertz (Hz). 9 Resistance—ohm (Ω). 10. Pressure—newton per square metre (N/m^2) 11. Potential difference—volt (V).

SECTION A

Experiment 1 :

(*a*) *To find the least count of a vernier callipers.*

(*b*) *To find the internal and external diameters and depth of a cylindrical body (say a calorimeter) with the help of a vernier callipers.*

(*c*) *To calculate the internal volume of a cylindrical body (say a calorimeter) and confirm the same using a graduated cylinder.*

Apparatus :

Vernier callipers, calorimeter and a graduated cylinder.

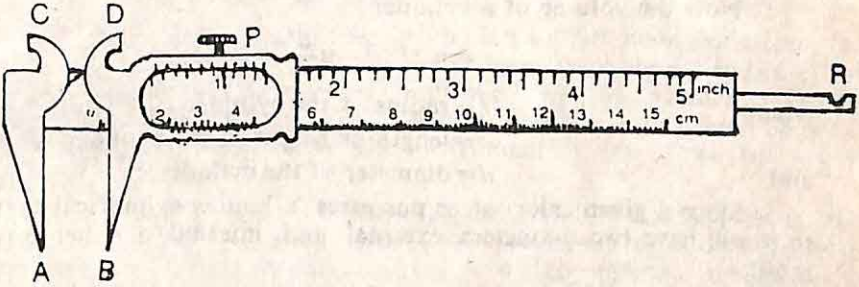


Fig. 1.1. Vernier callipers

Theory :

A very ingenious device for obtaining accuracy of a greater order than that obtainable by eye estimation was invented by P. Vernier (1580—1637) and is known by his name *i e.*, VERNIER. In this device a small auxiliary scale is provided, which slides along the ordinary scale, the divisions of this *Vernier Scale* being either a little longer or a little shorter than the divisions of the ordinary scale.

The great value of this device lies in its simplicity and in the fact that it can be used to measure to any fraction of a division required, if the auxiliary scale is divided suitably.

The auxiliary scale is graduated from a division which may be called the *zero of the Vernier*, this division being indicated by some distinguishing mark. The scale consists of n equal divisions on one side of the Vernier zero. These n Vernier divisions are exactly equal to $(n-1)$ scale divisions. Consequently one Vernier division is equal to $\frac{n-1}{n}$ or $\left(1-\frac{1}{n}\right)$ of a scale division. Thus, *each Vernier division is shorter than a scale division by* $\frac{1}{n}$ *of a scale division*. This quantity $\frac{1}{n}$ of a scale division is called the **least count** (or **Vernier Constant**) of the Vernier.

An instrument called the **Vernier Callipers** is used for measuring the linear dimensions of bodies. It consists of a metal rule furnished with two jaws, A and B (Fig. 1.1.) projecting at right angles to the rule. Of these A is fixed whilst the other B can slide backwards and forwards. The upper jaws C and D are used to measure the internal dimensions of the hollow objects. On the rule is engraved a scale divided into millimetres The sliding jaw B is also provided with a short scale V called a Vernier. The sliding strip or rod R is used to measure the depths of certain objects. It is connected to the Vernier scale and slides with vernier along the main scale.

Now the volume of a cylinder

$$=\pi r^2 . l=\pi\frac{d^2}{4} . l$$

where r=radius of the cylinder

l=length or height of the cylinder

and d=diameter of the cylinder.

Since a given calorimeter possesses a hollow cylindrical shape, so it will have two diameters external and internal and hence two radii.

If V_E is the external volume of the calorimeter, then

$$\boxed{V_E=\frac{\pi d_1^2}{4} . l_1}$$

where d_1=external diameter of the calorimeter

and l_1=external length of the calorimeter.

If V_i is the internal Volume of the calorimeter, d_2 is the internal diameter of the calorimeter and 'h' is the depth of the calorimeter, then

$$\boxed{V_i=\frac{\pi d_2^2}{4} . h}$$

Procedure :

(*i*) Determine the *least count* (or *vernier constant*) *i.e.*, the distance between the value of one main scale division and one vernier scale division of the vernier callipers as explained in the theory above.

(*ii*) Find the *zero error* by bringing the jaws A and B of the vernier callipers closely in contact. If there is no zero error this fact should be recorded.

The Zero Error :

In some instruments when the movable jaw B is brought in contact with the fixed jaw A, the zero of the vernier scale may not coincide with the zero of the main scale. In this case, the instrument is said to possess zero error which may be *positive* or *negative* according as the zero of the vernier scale lies to the right or left to the zero of the main scale and a correction has to be applied to the observed reading in order to determine the correct value.

To find the zero error, suppose the zero of the vernier scale lies to the right of the zero of the main scale, and *n*th vernier division coincides with any main scale division. The error is obtained by multiplying n with the vernier constant and a positive sign is put before it as the length measured by the instrument is more than the actual length.

If the vernier zero lies to the left of the main scale zero, the coinciding vernier division is to be seen from the other end of the vernier scale and its number multiplied by the vernier constant gives the required zero error with a negative sign placed before it as the length measured by the instrument is less than the actual length.

The zero error is always algebraically subtracted from the observed reading. Sometimes it is preferred to know the zero correction of the instrument. It has a sign opposite to that of the zero error but is equal in magnitude and is always algebraically added to the observed value to obtain a correct one.

(*iii*) To measure **external diameter** of the calorimeter, place it *diameterwise* between the two jaws A and B of the vernier callipers in such a way that it is just held between the jaws without undue pressure. Note the main scale reading which is just *before the zero* of the vernier scale and also find the number of the vernier division which is in line with a main scale division. Multiply this vernier division by the least count or vernier constant and add it to the main scale reading to get the observed external diameter of the calorimeter. The corrected external diameter is calculated by applying zero correction.

Repeat the observations *at least three times* at three different places in *two perpendicular directions*. The mean corrected external diameter (d) is found.

(*iv*) To measure **internal diameter** of the calorimeter, make use of the upper jaws C and D of the vernier callipers. Insert the upper jaws in the calorimeter and adjust the position of the movable jaw D in such a way that C and D touch the inner walls of the calorimeter gently without exerting undue pressure on them. Note the reading of the vernier callipers in the same manner as explained in (*iii*) above and then find the corrected internal diameter after applying zero correction.

Repeat the observations at least *three times* at three different places in *two perpendicular directions*. The mean corrected internal diameter (d_2) is found.

(v) To measure **depth** of the calorimeter, the sliding strip R of the vernier callipers (Fig. 1.1) is used.

Put the edge of the main scale of the vernier callipers on the mouth of the calorimeter (the callipers being held vertically), in such a way that the strip R is able to go inside the calorimeter along its length. Move the sliding jaw so that the strip R touches the bottom of the calorimeter gently.

Read the vernier callipers in the same manner as explained in (*iii*) above which gives the observed depth of the calorimeter. The corrected depth is calculated by applying zero correction.

Repeat the observations at least *three times* and then find the mean corrected depth (h) of the calorimeter.

(*vi*) Calculate the internal volume V_i by using the formula

$$V_i=\frac{\pi d_2^3}{4}\ h \text{ cm}^3.$$

(*vii*) Verify the internal volume thus calculated with the help of a graduated cylinder *i.e.*, fill the calorimeter upto the brim with water and then pour this water into the graduated glass cylinder and note the volume of water directly in cm^3.

Observations :

Least Count or Vernier Constant (V.C.)

$$n \text{ V.D.}=(n-1) \text{ M.S.D.}$$

where V.D. = Vernier scale division

and M.S.D. = Main scale division

$$1 \text{ V.D.}=\frac{(n-1)}{n} \text{ M.S.D.}$$

Difference [M.S.D. − V.D.]

$$=1-\frac{(n-1)}{n}\Big]$$

$$=\frac{1}{n} \text{ M.S.D.}$$

But 1 M.S.D. = x mm

Vernier Constant (V.C.)

$$=\frac{1}{n}\times x \text{ mm}$$

$$=\frac{x}{10n} \text{ cm}$$

For example :

$$10 \text{ V.D.} = 9 \text{ M.S.D.}$$

$$1 \text{ V.D.} = \frac{9}{10} \text{ M.S.D.}$$

Difference [M.S.D. − V.D.]

$$= \left(1 - \frac{9}{10}\right)$$

$$= \frac{1}{10} \text{ M.S.D.}$$

But 1 M.S.D. = 1 mm

∴ Vernier Constant (V.C.)

$$= \frac{1}{10} \times 1 \text{ mm}$$

$$= 0{\cdot}1 \text{ mm} = {\cdot}01 \text{ cm}]$$

Zero Error :

(1) ....cm (2) ....cm (3) ....cm

Mean Zero Error = ...cm

Mean Zero Correction = ±.........cm

External Diameter of the Calorimeter :

S. No.	One direction			Mutually perpendicular direction			Mean observed diameter in cm
	Main scale reading in cm (a)	No. of vernier division coincident (n)	Observed diameter in cm= (a)+n × V.C.	Main scale reading in cm (a)	No. of V.D. coincident (n)	Observed diameter in cm= (a)+n × V.C.	
1.							
2.							
3.							
						Mean = A =	......cm

Hence mean corrected

external diameter $= d_1 = \text{A} + $ (zero correction)

= ....cm

Internal Diameter of the Calorimeter :

S. No.	*On edirection*			*Mutually Perpendicular direction*			*Mean observed diameter in cm*
	Main scale reading in cm (a)	*No. of vernier division coincident (n)*	*Observed diameter in cm= (a)+n × V.C.*	*Main scale reading in cm (a)*	*No. of V.D. coincident (n)*	*Observed diameter in cm= (a)+n × V.C.*	
1.							
2.							
3.							

Mean = B = ...cm

Hence mean corrected

internal diameter $= d_2 = B +$ (zero correction)

$= \ldots\ldots\ldots\ldots$ cm

Depth of the Calorimeter :

S. No.	*Main scale reading in cm (a)*	*No. of V.D. coincident (n)*	*Vernier scale reading in cm $= n \times V.C.$*	*Observed depth in cm=(a) $+ n \times V.C.$*	*Mean observed depth in cm*
1.					
2.					
3.					

Hence mean corrected

depth of the calorimeter

$(h) =$ (Mean observed depth) $-$ (zero correction)

$= \ldots\ldots\ldots$ cm

Calculations :

Internal Volume of the Calorimeter

$$V_i = \frac{\pi d_2^2}{4}\, h = \ldots\ldots\ldots \mathrm{cm}^3$$

$$= \ldots\ldots\ldots \mathrm{m}^3$$

Verification by Graduated Cylinder :

Volume of water filling the calorimeter upto the brim

$$= \ldots\ldots\ldots\ldots \mathrm{cm}^3$$

$$= \ldots\ldots\ldots\ldots \mathrm{m}^3$$

Result :

(*i*) Least count of the vernier callipers = ...cm

(*ii*) External diameter of the calorimeter =........cm

(*iii*) Internal diameter of the calorimeter = ... cm

(*iv*) Depth of the given calorimeter = ...cm

(*v*) Internal volume of the given calorimeter = ...cm^3

= ....m^3

Precautions :

(*i*) The zero error should be noted carefully with *sign* and taken into account.

(*ii*) The jaws should not be *pressed* too hard.

(*iii*) The dimension to be measured (length, diameter or depth) should be parallel to the main scale.

(*iv*) The calorimeter should be held between those positions of the jaws which are in *contact* when the callipers is closed without placing the calorimeter between them.

(*v*) The calorimeter between the jaws should not be held either too tight or too loose.

(*vi*) Oil the vernier if its motion is not smooth.

(*vii*) While measuring the depth, the strip should be perpendicular to the bottom surface.

(*viii*) While measuring the depth, the edge of the main scale should not get out of contact from the mouth of the calorimeter when the end of the sliding rod R touches the bottom.

(*ix*) Zero error for the measurement of depth is different from the usual zero error of a vernier callipers. So it should be determined separately and carefully as explained below :

[When the two jaws A and B are made to touch each other, the end of the sliding strip R and the edges of the main scale get in line with each other in a perfect instrument. In a defective instrument, the end of the strip R may not be exactly in line with the edge of the main scale when the instrument reads zero. In such a case, it is said to possess *zero error which is relevant only for depth measurement*. The reading when the end of the sliding strip R lies exactly in line with edge of the main scale, equals the *magnitude* of this zero error.]

Sources of Error :

1. Vernier jaws may not be at right angles to the main scale or parallel to each other
2. None of the vernier divisions may be exactly coincident.
3. The vernier scale may be loose.
4. The graduations on the scales may not be evenly marked.

Exercises

(1) *To find (a) the diameter (b) the surface area, and (c) the volume of a sphere using vernier callipers.*

Hint : Measure the diameter of the sphere at four different places as in Expt. 1 and calculate its surface area and volume.

Surface area of sphere $= 4\pi r^2$

Volume of sphere $= \frac{4}{3}\pi r^3$

(2) *To determine the internal and external diameter of a calorimeter and to calculate the thickness of its wall. Also find the depth of the calorimeter with the help of vernier callipers.*

Hint : Measure the external diameter by using the jaws A and B and internal diameter by using the jaws C and D. Difference of the two gives *twice* the thickness of the calorimeter wall.

(3) *To determine the length and diameter of a given cylinder by vernier callipers and hence calculate its volume.*

ORAL QUESTIONS

Q. 1. *What is a vernier callipers? Why is it called a vernier callipers?*

Ans. It is an instrument used to measure small lengths and diameters accurately. It is called vernier callipers after the name of its inventor "Pierre Vernier".

Q 2 *What is Least Count?*

Ans. It is the least distance that can be measured by the instrument.

Q. 3. *Define vernier constant.*

Ans. It is the difference between the smallest division of main scale and that of vernier scale.

Q. 4. *What part of the instrument is the vernier seale?*

Ans. The small scale that slides along the main scale is called vernier scale.

Q 5. *What is the difference between Vernier Constant and Least Count?*

Ans. Practically no difference. Vernier constant is the Least count but is used only for vernier callipers whereas least count is used for all the instruments.

Q. 6. *What is zero error ?*

Ans. The instrument is said to have zero error, if the zero of the main scale does not coincide with the zero of the vernier scale when the two jaws of the vernier callipers are brought into contact.

Q. 7. *When is the zero error positive ? When is the zero error negative ?*.

Ans. If the zero of the vernier scale is towards the right of the main scale, the error is said to be positive (Fig. 1.2). If the zero of the vernier scale is towards the left to the main scale, the error is said to be negative (Fig. 1.3).

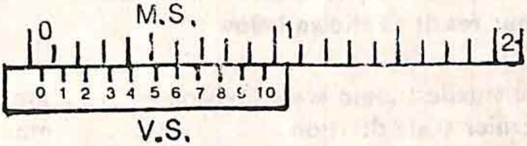


Fig. 1.2. Positive error

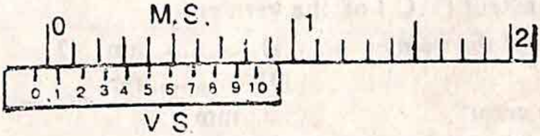


Fig. 1.3. Negative error

Q. 8. *What is zero correction ?*

Ans. It is equal in magnitude but opposite in sign to zero error.

Q. 9. *Why does zero error creep in ?*

Ans. It creeps in because of wear and tear due to the long use of the instrument.

Q. 10. *What is 'angular vernier' ?*

Ans. Angular vernier is used to measure fractions of a degree. These are provided in spectrometers and sextants.

Q. 11. *Which will be more accurate—a calliper of small V.C. or that of large V.C. ?*

Ans. A calliper of small V.C.

Q. 12. *What is the function of the upper jaws of the vernier callipers ?*

Ans. To measure the internal dimensions of the hollow objects.

Q. 13. *What is the function of the sliding strip or rod ?*

Ans. To measure the depths of certain objects.

Experiment 2 :

To find the volume of the given metallic rectangular block by measuring its dimensions with a vernier callipers and check up the result by using a graduated cylinder.

Apparatus :

Metallic block ; vernier callipers ; graduated cylinder.

Procedure :

Measure the length, breadth and thickness of a given metallic rectangular block using a vernier callipers **as explained in Experiment 1.** Measure the length, breadth and thickness of the block about 4 to 8 times in various positions of the block.

Tabulate your result as shown below :

Observations :

Value of the smallest main scale division = ...mm

Size of a vernier scale division = ...mm

Least count of the vernier Or Vernier constant (V.C.) of the vernier = ...mm

Zero error of the vernier = (1) ....mm (2) ....mm (3) ....mm

Mean zero error = ...mm

Mean zero correction = ± ....mm

Length of the block :

S.N.	*Main scale reading in mm (a)*	*No. of vernier division coincident (n)*	*Length of the block in mm*		*Mean corrected length of the block (l) in mm*
			Observed = (a) + n × V.C.	*Corrected = Observed ± Mean Zero correction*	
1.					
2.					
3.					
⋮					
8.					

Breadth of the block :

S.N.	*Main scale reading in mm (a)*	*No. of Vernier division coincident (n)*	*Breadth of the block in mm*		*Mean corrected breadth of the block (b) in mm*
			Observed= (a)+n×V.C.	*Corrected =Observed ±Mean Zero correction*	
1.					
2.					
3.					
⋮					
8.					

Thickness of the block :

S.N.	*Main scale reading in mm (a)*	*No. of Vernier division coincident (n)*	*Thickness of the block in mm*		*Mean corrected thickness of the block (t) in mm*
			Observed= (a)+n×V.C.	*Corrected =Observed ±Mean Zero correction*	
1.					
2.					
3.					
⋮					
8.					

Calculations :

Volume of the given metallic block

$=$ Mean corrected length $\times$ Mean corrected breadth $\times$ Mean corrected thickness

$= l \times b \times t$

$=$ ....$mm^3 =$ ....cm^3

Verification by graduated cylinder :

Volume of water in the graduated cylinder before immersing the block in water in the cylinder $= A =$ ....cm^3

Volume of water in the graduated cylinder after immersing the block in water in the cylinder $= B =$ ....cm^3

∴ Volume of the given metallic block $= B - A$............cm^3

Precautions and Sources of Error :

Same as in Expt. 1.

ORAL QUESTIONS

Same as in Expt. 1.

Experiment 3 :

(*a*) To find the least count of a screw gauge.

(*b*) To measure the diameter of a given wire by using screw gauge and compare it by wrapping the same wire around a pencil and explain the difference.

(*c*) To find the volume of the given wire by using screw gauge.

(*d*) To find the gauge of the wire using standard table.

(*e*) To find the thickness of a given glass strip using screw gauge.

Apparatus :

A screw gauge ; a wire and a glass strip.

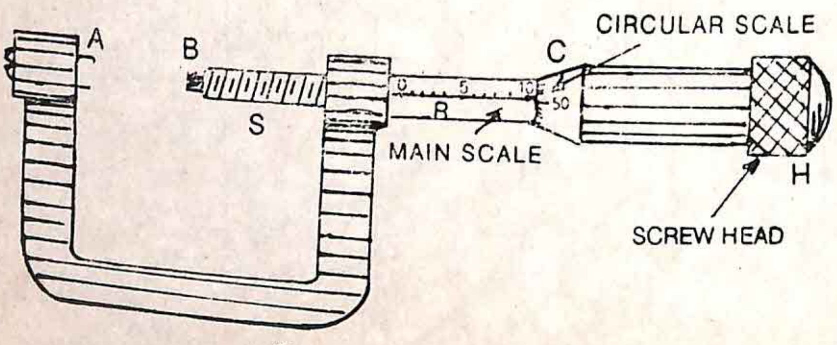


Fig. 3.1. Screw Gauge

Procedure :

(*i*) *Determine the least count* of the instrument. In order to do that bring the zero mark of the circular scale into coincidence with the reference line. Note the reading on the main scale. Give some complete rotations say four or five to the circular scale and note the reading on the main scale again. Difference of the two readings gives the distance moved by the screw. Calculate the pitch by the relation.

$$\text{pitch} = \frac{\text{distance moved by the screw}}{\text{number of rotations}}$$

Note the total number of divisions on the circular scale, then

$$\text{least count} = \frac{\text{pitch}}{\text{total number of divisions on the circular scale}}$$

(*ii*) *Find the zero correction i.e.*, screw up until the gap between A and B is just closed and the ratchet arrangement gives a click. If the zero division of the circular scale coincides with the reference line, write the zero error as nil ; otherwise reference line may be 'x' divisions towards positive side of circular scale, then

$$\text{zero error} = +x \times \text{L.C.}$$

and $$\text{zero correction} = -x \times \text{L.C.}$$

And if the reference line may be 'x' divisions towards negative side of zero of the circular scale then

$$\text{zero error} = -x \times \text{L.C.}$$

and $$\text{zero correction} = +x \times \text{L.C.}$$

(*iii*) *Diameter of the Wire*. In order to find the diameter of a given wire, place it in the gap between A and B. Turn the screw head until the ratchet arrangement gives a click. Note down the reading on the main scale. Read the division of the circular scale coinciding with the reference line. Multiply it by the least count and add it to the number of complete divisions visible on the main scale, *i e.*, the reading on the main scale.

Turn the wire through a right angle and note the reading again at the same point.

Repeat the observations at least five times, take the mean and correct the reading by applying zero correction.

(*iv*) Remove the kinks in the wire if there is any. Wrap about 40 turns of this wire on a pencil so as to form a coil having its turns very close to each other as shown in Fig. 3 2. Measure the length (t) of the coil with the help of mm scale.

Count the number of turns (n) wrapped as well. Then diameter of the given wire by wrapping $= \frac{t}{n}$.

(*v*) Consult the standard table of (S.W.G.) for finding the gauge of the wire corresponding to its calculated diameter.

(See Appendix Table)

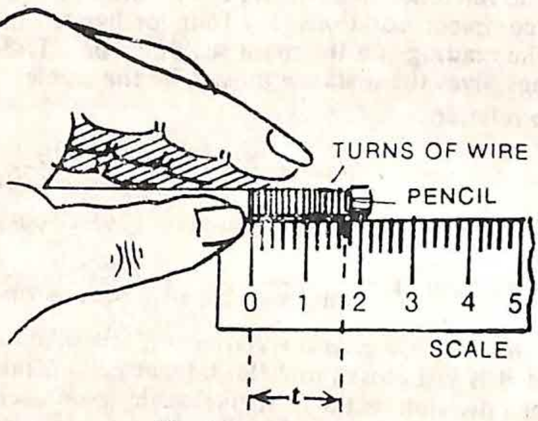


Fig. 3.2

(*vi*) **Measurement of Thickness.** Place the glass strip in the gap between A and B. Turn the screw head until the ratchet arrangement gives a click. Note down the reading on the main scale. Read the division of the circular scale coinciding with the reference line. Multiply it by the least count and add it to the main scale reading.

Repeat it three times by placing the glass strip in different positions.

Observations and Calculations :

No. of rotations given to the screw = ...

Distance moved on the main scale = ... mm

$$\text{Pitch} = \frac{\text{distance moved on the scale}}{\text{number of rotations}}$$

Total number of divisions on the circular scale = ...

$$\text{Least count (L.C.)} = \frac{\text{Pitch}}{\text{no. of divisions on the circular scale}}$$

Zero error $= \pm x \times \text{L.C.}$

Zero correction $= \mp x \times \text{L.C.} =$ ....cm

Diameter of the Wire :

S. No.	*One direction*			*Mutually perpendicular direction*			
	Main scale reading (a) in mm	*No. of circular division coinciding (m)*	*Observed diameter in mm= $a+m \times L.C.$ (d_1)*	*Main scale reading (a) in mm*	*No. of circular division coinciding (m)*	*Observed diameter in mm= $a+m \times L.C.$ (d_2)*	*Mean observed diameter $d=\left(\frac{d_1+d_2}{2}\right)$ in mm*
1.							
2.							
3.							
4.							
5.							
							Mean= ....mm

Mean Corrected diameter $=(d)=$ ... .mm

$=$ ....cm

The **gauge** of the wire from a standard table= ....

Diameter of the wire by wrapping :

Number of turns $=n=$ ....

Length of the coil $=t=$ ....cm

$\therefore$ Diameter of the wire $=\frac{t}{n}=$ ....cm

Volume of the wire :

Mean corrected radius $=r=\frac{d}{2}=$ ....cm

Length of the wire $=t=$ ....cm

Volume of the wire $=V=\pi r^2 . t=$ ....cm^3

Thickness of the Glass Strip :

S. No.	*Main Scale reading (a) in mm*	*No. of circular division coinciding (m)*	*Observed thickness $t=a+m\times L.C.$ (mm)*
1.			
2.			
3.			

Mean observed thickness 't'= ....mm

Mean corrected thickness 't'= ....cm

Result :

(*i*) Diameter of the wire by screw gauge= ....cm

(*ii*) Diameter of the wire by wrapping method= ....cm

(*iii*) Volume of the wire= ....cm^3

(*iv*) Gauge of the wire= ....cm

(*v*) Thickness of the glass strip= ....cm

Precautions :

1. Undue pressure should not be applied to bring the two studs in contact. For this, the screw should always be turned by the ratchet cap.

2. The screw should move freely without any friction.

3. The zero error, if any, should be noted with proper sign and taken into consideration.

4. The screw should always be moved in the *same* direction to avoid back-lash error.

5. Readings should be taken at different places along the length of the wire, taking two readings at *right angles* to each other at the same place.

Sources of Error :

The turns of the coil formed of the wire may not be in close contact with each other.

Exercises

Q. 1. *Find the thickness of a small coin.*

Q. 2. *Find the volume of a rectangular glass plate using a vernier callipers and a screw gauge ?*

Hint. Volume = Length × Breadth × Thickness.

ORAL QUESTIONS

Q. 1. *What do you mean by screw gauge ?*

Ans. It is an instrument used to measure very short lengths accurately such as thickness of paper, diameter of wire, etc.

Q. 2. *Why screw gauge is called Micrometer ?*

Ans Because it can measure distance of the order of one millionth part of a metre, *i.e.*, 10^{-6} m *i e.*, the least count of the screw is of the order of a micrometer.

Q. 3. *Why it is called a screw gauge ?*

Ans. Because it is based on the principle of a screw and measures, *i.e.*, gauges diameter of the wire.

Q. 4. *What is the principle of a screw gauge ?*

Ans. Its principle is that linear motion is directly proportional to the rotation given to the head.

Q. 5. *What do you mean by pitch of the screw ?*

Ans. It is the linear distance moved when its head is given one complete rotation

Q. 6. *How is the least count of the screw determined ?*

Ans. It is determined by dividing the pitch of the screw by number of divisions on the circular scale.

Q. 7. *Is pitch related with least count ?*

Ans. Yes ;

$$\text{L.C.} = \frac{\text{Pitch}}{\text{no. of divisions on the circular scale}}$$

Q. 8. *What is zero-error ? How do you find its sign ?*

Ans. When the screw is in contact with plane stud and the zero of the circular scale does not coincide with the zero of the main scale, the instrument is said to have zero-error.

When the screw is in contact with the plane stud opposite to it and if the zero of the circular scale is below the reference line, the error is said to be positive (Fig. 3.3). If the zero of the circular scale is above the reference line, the error is said to be negative (Fig. 3.4).

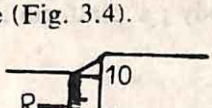


Fig. 3.3. Positive error

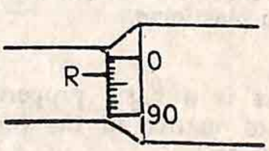


Fig. 3.4. Negative error

Q. 9. *What is zero-correction ? How is it related with zero-error ?*

Ans. Zero-correction is numerically equal to zero-error with opposite sign.

Q. 10. *What is back-lash error ? How is it avoided ?*

Ans. This error is associated with all instruments working on screw nut principle. The error is developed due to wear and tear of the screw with use. Due to looseness between the screw and the nut, the screw does not move backward or forward for a little motion of the nut (or cap) in the opposite direction. Such an error is called back-lash error.

It can be avoided by turning the screw in one and the same direction while taking all observations.

Q. 11. *What is the advantage of taking two observations mutually perpendicular to each other at each point while measuring the diameter of wire ?*

Ans. To take into account any deviation in the cross-section of the wire from its perfectly circular shape.

Q. 12. *Which is more accurate a vernier callipers or a screw gauge and why ?*

Ans. Screw gauge.

With the help of vernier callipers we can measure accurately up to 0·1 mm while with a screw gauge we can measure lengths with greater accuracy upto $\frac{1}{100}$th of a mm.

Q 13. *Are the values of diameter of a wire obtained by screw gauge and by wrapping method the same ?*

Ans. No ; they are different.

Q. 14. *Which method gives more accurate value and why ?*

Ans. Screw gauge method. The accuracy of measurement by an instrument depends upon the least count of the instrument. Since the least count of screw gauge is $\frac{1}{100}$th that of a metre scale used in wrapping method, so the value of the diameter obtained by screw gauge is far more accurate.

Experiment 4 :

To determine the inertial mass of a body.

Apparatus :

A G-clamp stand, two hacksaw blades or two steel strips ; set of known masses (a weight box) ; a given body ; a platform ; a stop watch and plasticine.

Theory :

Mass is a basic property of matter and is a measure of the quantity of matter in the body. The S.I. unit of mass is the kilogram (kg).

Newton's first law of motion leads us to the concept of inertia, a property of everybody. Newton's second law of motion proceeds a step further and provides us with a quantitative measure of inertia namely mass. *Mass as defined by Newton's second law relates the*

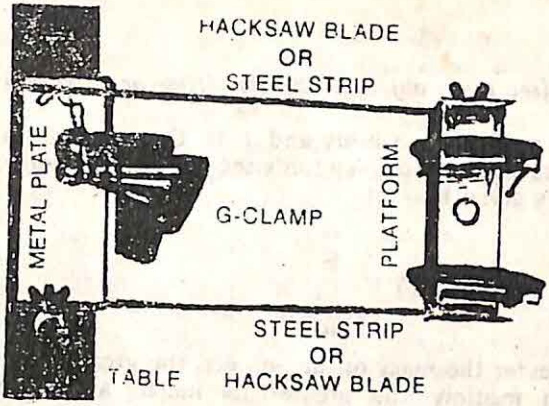


Fig. 4.1. Inertia Balance.

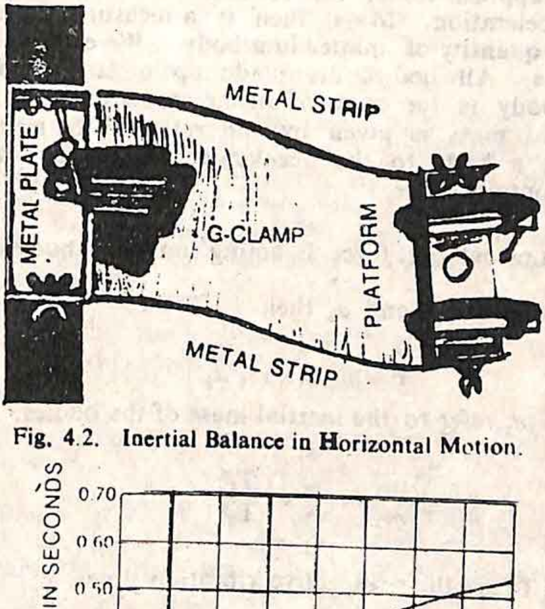


Fig. 4.2. Inertial Balance in Horizontal Motion.

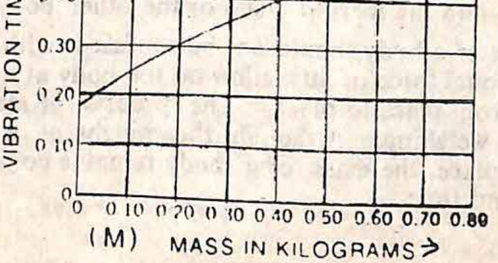


Fig. 4.3.

net force applied to an object to the resulting acceleration. If $\vec{F}$ is the net force applied on a body and $\vec{a}$ is the acceleration produced in it, then, according to Newton's second law of motion, the mass of the body is given by

$$m = \frac{\vec{F}}{\vec{a}}$$

The greater the mass of an object, the greater its resistance to a change in motion—the greater its inertia and in terms of the second law of motion, we see that the greater the mass of an object for a given applied force, the smaller the change in motion—the smaller its acceleration. Mass, then is a measure of inertia and is related to the quantity of matter in a body. *We call such a mass as* **inertial mass**. All bodies are made up of atoms. The inertial mass of a body is the sum of all the atoms put together in that body. Inertial mass is given by the ratio of the resultant force applied upon a body to the acceleration produced in it along a smooth horizontal surface.

If the same external force $\vec{F}$ acting on two bodies produces in them acceleration $\vec{a}_1$ and $\vec{a}_2$ then

$$\vec{F} = m_1 \vec{a}_1 = m_2 \vec{a}_2$$

where m_1 and m_2 refer to the inertial mass of the bodies.

$$\therefore \quad \frac{m_1}{m_2} = \frac{\vec{a}_2}{\vec{a}_1} = \frac{T_1^2}{T_2^2}$$

where T_1 and T_2 are their respective vibration times.

If one of the bodies is taken as a standard, the above equation can be used to find the inertial mass of the other body.

The mass of a body should not be confused with weight which is the gravitational force of attraction on the body at a certain place and it differs from place to place. The process of measurement of mass is called weighing. Although the weight of a body differs from place to place, the mass of a body remains constant irrespective of place and time.

Procedure :

1. Fix the two hacksaw blades or two steel strips as shown in Fig. 4.1. *The 'hacksaw' blades (or similar metal strips) should be*

Date 31.5.06
Accn. No.... 12194

clamped tightly between the small metal plates using a G-clamp. It is very important to have the blades held tightly and symmetrically just where the blades emerge from the jaws as otherwise energy is lost too fast and the oscillations are badly damped. The whole set up is clamped to the edge of the table as shown in Fig. 4.1. At the free ends of the blades attach a small platform or a pan made of card board or wood. It is to be used for holding the body or a known mass.

2. Push the platform or the pan to one side with a finger and release it so that the platform starts oscillating in a *horizontal plane* Fig. 4.2. *Since the blades vibrate horizontally the action of the inertia balance is entirely independent of gravity*, which acts vertically. Make sure that the blade oscillates for about 10 oscillations with almost the same amplitude. This is necessary for a satisfactory experiment.

Note : The period of vibration can be increased by reducing the length of the blades protruding from the jaws.

3. Load the platform with a known mass of say 10 g. Note that the mass of the body loaded on the platform is assumed to be inertial mass. Make the platform to oscillate with a suitable amplitude *so that the mass placed on the platform does not slide when it oscillates.* For ensuring this, it is better to initially put a thin layer of plasticine on the platform. The mass placed on it will stick and will not slide.

Find out the time (t) taken for 10 oscillations by using a stop-watch and calculate the period of one oscillation, called the time period (T).

4. Repeat the activity for at least five known masses by loading the platform with each of these in turn.

5. Plot a graph between the total mass (m) of the platform and the object placed on it and its vibration time (T). Fig. 4.3 shows a graphical plot of one such set of observations. The graph in Fig. 4.3 is not a straight line. But by careful analysis of the data, the relationship can be expressed by the following equation :

$$\frac{m_1}{m_2}=\frac{T_1^2}{T_2^2}$$

in which m_1 and m_2 are two masses (*including* the mass of the platform or pan in each case) and T_1 and T_2 are their respective vibration times.

6. Place the body of *unknown* mass (M) on the platform. Let the platform oscillate *horizontally* as before. Measure the time period T′. For this value of T′, find out the total mass m' from m versus T graph as shown in Fig. 4.3.

Observations and Calculations :

Mass of the platform or the pan $= M_1 =$ ......g

S. No.	*Known mass placed on the platform or the pan M_2 in g*	*Total mass $(m) = (M_1+M_2)$ in g*	*Time taken (t) for 10 oscillations in sec*	*Time. period $(T) = \frac{t}{10}$ in sec*
1.				
2.				
3.				
4.				
5.				
6.				
7.				

For the unknown inertial mass :

Time (t') taken for 10 oscillations in sec	*Time. period $T' = \frac{t'}{10}$ in sec*

From m versus T graph, the total mass (m') of the unknown object and the platform as read against

T' = Intercept of the graph on the abscissae

= ....g

Inertial mass (M) of the body

$= m' - M_1)$

= ....g

Result

Inertial mass of a given body

=.........g

Precautions :

1. The position of the blades of the inertial balance should remain horizontal when the platform at its free ends, is loaded with known masses. To ensure this, *fix the blades tightly at the clamped end otherwise there will be energy losses and the oscillations are badly damped.*

2. Oscillate the blade with a suitable amplitude, so that the mass placed on the platform does not slide when it oscillates. For ensuring this, it is better to initially put a thin layer of plasticine on the platform. The mass placed on it will stick and will not slide.

3. Place each body at the centre of the platform so that it does not have a tendency to twist the blade.

4. Attach the platform at the free ends of the two hacksaw blades/strips in such a way that centre of gravity of the body placed on it is as close to the horizontal plane bisecting the blades, as possible.

5. The number of times the inertial balance vibrates in a second of time depends upon the *length* and *stiffness* of the two supporting metal blades and upon the total mass of the pan and any objects placed on it. The more *massive* the pan and its contents, the *slower* will be the changes in their motion and the *longer* will be the time they will take to go through a complete vibration.

ORAL QUESTIONS

(Same as in Expt. 5)

Q. 1. *What is inertia ?*

Ans. By inertia we mean the inertness *i.e.*, built in reluctance of matter and hence its ability to change its position of rest or of uniform motion in a straight line without the application of an external force.

Q. 2. *How is inertia related to the mass of a body ?*

Ans. The mass of a body is a measure of its inertia. A large mass requires a large force to produce a certain acceleration because the greater the mass of the body the greater is its tendency to oppose any change in its state of rest (*inertia of rest*) or of uniform motion (*inertia of motion*).

Q. 3. *What is inertial mass ?*

Ans. Mass (*m*) as defined by Newton's second law of motion relates the net force $(\vec{F})$ applied to an object to the resulting acceleration $(\vec{a})$.

$$m=\frac{\vec{F}}{a}$$

The greater the mass of an object, the greater its resistance to a change in motion—the greater its inertia. We call such a mass as *inertial mass*. It is given by $\frac{F}{a}$

Q. 4. *What is gravitational mass then?*

Ans. The mass of a body which determines the gravitational pull due to the earth acting upon it is called its gravitational mass.

Q. 5. *Is there any difference between the inertial mass and the gravitational mass of a body?*

Ans. Experiments have failed to show any difference. Experiments show that these two masses are *equal* for a given body.

Experiment 5 :

To find the density of a substance of the given rectangular block using a beam balance.

Apparatus :

Rectangular iron/wooden block ; vernier callipers ; physical balance and a weight box.

Diagram :

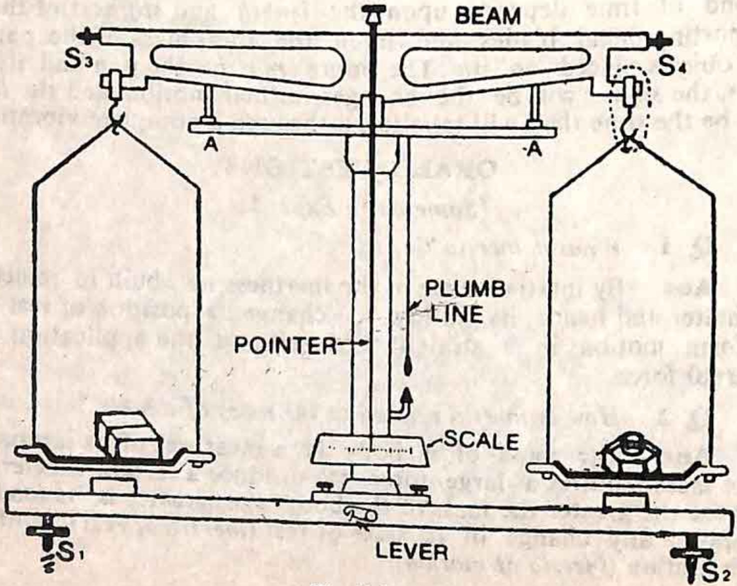


Fig. 5.1

Theory :

If we take several solid blocks of wood or iron, we find that the larger a block is, the larger is its mass. If, however, the mass of each block is divided by its volume, the result in each case is the same which is characteristic of the substance of which the blocks are made. This result is called density.

$$\therefore \quad \text{Density of a substance } (\rho) = \frac{\text{Mass of the body}}{\text{Volume of the body}}$$

$$= \frac{M}{V}$$

The S.I. unit of density is kg m^{-3}.

Procedure :

1. Measure the length, breadth and thickness of the given rectangular block by using the vernier callipers as explained in Experiment 2 and hence calculate its volume.

2. Check the setting of the physical balance carefully *i.e.*, its pillar is vertical and when raised, its pointer swings equally on both sides of zero mark and swings without jerks.

3. Keep the rectangular block in the left-hand pan when the beam is in the arrested position. Counter poise, carefully by loading/unloading in steps the right-hand pan with known masses from the weight box. Use forceps for handling the known masses.

4. Calculate the density of the rectangular block using the formula

$$\rho = \frac{\text{mass of the rectangular block}}{\text{its volume}}$$

Observations :

Value of the smallest main scale division = ...mm

Size of a Vernier scale division = ...mm

Vernier constant (V C.) of the vernier = ...mm

Zero error of the vernier

= (1) ....mm (2) ....mm (3) ....mm

Mean zero error = ...mm

Mean zero correction = ± ......mm

Length of the block :

S.No.	*Main Scale Reading in mm (a)*	*No. of Vernier Scale coincident (n)*	*Length of the block in mm*		*Mean corrected length (l) of the block in mm*
			Observed= (a)+n × V.C.	*Corrected =Observed ±Mean Zero correction*	
1.					
2.					
3.					

Breadth of the block :

S.No.	*Main Scale Reading in mm (a)*	*No. of Vernier Scale coincident (n)*	*Breadth of the block in mm*		*Mean corrected breadth (b) of the block in mm*
			Observed= (a)+n × V.C.	*Corrected =Observed ±Mean Zero correction*	
1.					
2.					
3.					

Thickness of the block :

S.No.	*Main Scale Readnig in mm (a)*	*No. of Vernier division coincident (n)*	*Thickness of the block in mm*		*Mean corrected thickness (t) of the block in mm*
			Observed= (a)+n×V.C.	*Corrected =Observed ±Mean Zero correction*	
1.					
2.					
3.					

Mass of the rectangular block by
physical balance $=M=$.........g
$=$.........kg

Calculations :

Volume (V) of the rectangular block

$$= l \times b \times t$$

$=$.........mm^3

$=$.........cm^3

$=$.........m^3

Density of the substance of the given

$$\text{rectangular block} = \rho = \frac{\text{mass of the block}}{\text{volume of the block}}$$

$$= \frac{M}{V}$$

$=$.........g/cm^3

$=$.........kg m^{-3}

Result :

Density of the substance of the given
rectangular block$=$.........kg m^{-3}

Precautions and Sources of error :

1. As regards vernier callipers, the precautions and sources of error to be observed are the same as given in Experiment 1.

2. Make sure that the scale pans of the physical balance are clean, dry and dust free as the mass of the dust may be more than its sensitivity.

3. Adjust the screws at the base of the physical balance until the plumb line hangs vertically over the fixed pointed tip.

4. Use a forcep to handle the weights. Never leave weights lying about.

5. While weighing always lower the case of the balance box to prevent disturbance due to air draughts.

6. Every balance has a certain capacity *i.e.*, maximum mass which can be placed on the pans and measured with precision. Do not over load it Find its capacity for measuring mass from its manufacturer's specifications.

ORAL QUESTIONS

(*Also read Questions of Expt. 1*).

Q. 1 *Define mass of a body. What is its unit ?*

Ans. The mass of a body is defined as the Quantity of matter contained in it. According to Newton's First Law of Motion, the mass of a body may also be defined as the quantity of inertia possessed by the body which makes it to resist any change in its state of rest or motion

The unit of mass is *kilogram* in SI system.

Q. 2. *Define density of a body.*

Ans. The density of a substance is defined as the mass per unit volume of that substance. This can be written as

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

Q. 3. *What is the unit of density ?*

Ans. The unit of density in SI system is *kilogram per cubic metre* or *kg*$^{-3}$.

Q. 4. *Define relative density of a substance ?*

Ans. Relative density of a substance is defined as the ratio of density of the substance to the density of water.

Q. 5. *What is the unit of Relative density ?*

Ans. The relative density is a number and hence it has no unit

Q. 6 *What happens to the weight of a body when immersed in water ?*

Ans. When a body is immersed in water, an upward thrust acts upon it. Due to this upward thrust, there is a loss in the weight of the body. This loss in weight of the body is equal to the weight of the water displaced.

Q. 7. *How does density vary with temperature ?*

Ans. It decreases with the increase of temperature.

Q. 8. *What is the density in SI units of (i) Water, (ii) Mercury and (iii) Ice ?*

Ans. (*i*) 1000 kg/m^3 at 4°C

(*ii*) 13600 kg/m^3

(*iii*) 930 kgm^{-3}

Q. 9. *Is relative density different from specific gravity ?*

Ans. No. They are the same terms.

Q. 10. *Is the relative density of a body different in different systems ?*

Ans. No. It is the same in all systems.

Q. 11. *What is the apparent weight of a block of wood floating on water ?*

Ans. Zero.

Q. 12. *Will the buoyancy be the same for a wooden block and an iron block of the same size pushed under the surface of water ?*

Ans. Yes ; the same.

Q. 13. *Why can't we extinguish burning kerosene oil by pouring water on it ?*

Ans. Kerosene oil is lighter than water. When water is poured over it, it sinks and, therefore, cannot prevent the air (responsible for burning) from coming in contact with the oil.

Q. 14. *A person carries a bucket of water weighing 20 kg in his right hand and a fish weighing 20 kg in the other. He puts the fish in the bucket. How much does his right hand now carry if the relative density of the fish is 1 ?*

Ans. His right hand would carry the same weight 20 kg as before. The addition of the fish will not add anything to the weight because the weight in water will be zero, its specific gravity being one.

Q. 15. *A piece of ice is floating on water in a beaker. When it melts completely, will the level of water go up, go down or remain the same ?*

Ans. It will remain the same.

Q. 16. *What is a Physical Balance ? What is its principle ?*

Ans. It is a lever of first order having the effort arm and the weight arm equal. It is based on the principle of moments.

Q. 17. *Why are knife edges made of agate ?*

Ans. Agate is a very hard substance and does not wear out easily, hence it is used.

Q. 18. *What is the function of Plumb Line ?*

Ans. To make the beam rotate in a vertical plane.

Q. 19. *Why are there only three levelling screws in the balance ?*

Ans. Because any three points can determine a plane, so the balance can be set with its beam horizontal.

Q. 20. *What are the requisites of a good balance ?*

Ans. A good balance should be true, sensitive and stable.

Q. 21. *What is the difference between mass and weight ?*

Ans. Mass of a body is the quantity of matter contained in a body and weight is the pull exerted by the Earth on the body.

Q. 22. *What does the Physical balance measure, weight or mass and why ? Name the instrument for measuring weight.*

Ans. We measure mass of a body with a Physical Balance as the acceleration due to gravity on both the pans, being equal, will cancel out and the readings will be independent of '*g*'.

Weight is obtained by spring balance.

Q. 23. *What do you understand by the sensitiveness of a balance and how is it increased ?*

Ans. A balance is said to be sensitive when its beam is turned through an appreciable angle for a small difference of weights in the two pans. The conditions of sensitiveness are :

(*i*) the arms of the pans must be large.

(*ii*) the beam must be light.

(*iii*) the distance between the C.G. of the beam and its fulcrum, *i.e.*, the central knife edge must be small.

Q. 24. *How stability is opposed to sensitiveness ?*

Ans. A balance is said to be stable if after a small displacement it comes back to its original position as quickly as possible.

The necessary conditions for stability are :

(*i*) the arms of the beam must be small.

(*ii*) the beam must be heavy.

(*iii*) the distance between the C.G of the beam and its fulcrum must be large

Thus sensitive balance cannot be stable and *vice versa*

Q. 25. *What is the principle of a spring balance ?*

Ans. The extension produced in the spring is directly proportional to the applied force provided the force applied is not too large to stretch the spring beyond elastic limits (Hooke's law).

When the spiral spring is subjected to a tension along the axis of a spiral in a spring balance, the movement of the index along the scale (indicating the elongation of the spring) being accurately proportional to the applied force.

Q. 26. *Can a spring balance measure mass ?*

Ans. Though primarily used to measure the weight of a body, it can also measure mass accurately *but only at the place of graduation.*

Q. 27. *Define sensitiveness of a spring balance.*

Ans. It is the change in length of the spring of a balance for a *unit change* in the weight hung.

Q. 28. *How can you increase the sensitiveness of a spring balance ?*

Ans. By making the spring of the balance long (as the change in length will be large when the original length of the spring is large).

Q. 29. *Will a spring balance calibrated in India measure the weight of a body accurately in France ?*

Ans. Yes ; it will measure weight accurately.

Q. 30. *What is Archimede's principle ? To what use it has been put ?*

Ans. Archimede's Principle : When a body is wholly or partially immersed in a fluid at rest, it experiences an upward thrust which is equal to the weight of the fluid displaced by it.

It is used to determine the relative densities of solids and fluids.

Experiment 6

To find the least count of the given stop-clock / watch and use it for checking the time period of oscillations of varying amplitude of a simple pendulum.

Apparatus :

Laboratory stand; split cork; fine cotton thread; a pendulum bob; stop-watch; card board; pencil : protractor.

Theory :

Same as in Expt. 16.

[For angles less than 15°, the error introduced by assuming that sin $\theta \approx \theta$ is only about 1 percent. Therefore for pendulum vibrations in which the amplitude of the motion does not exceed 15°, pendulum motion is a very good approximation to S.H.M.]

Diagram :

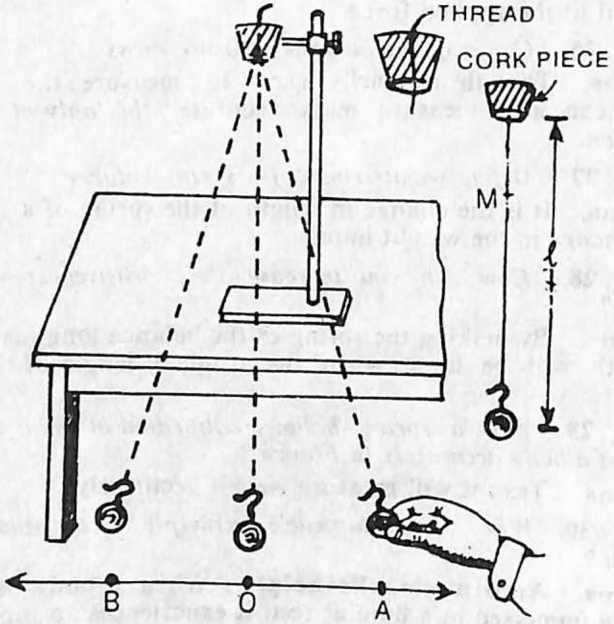


Fig. 6.1. Simple Pendulum.

Procedure :

1. Tie one end of the thread to pendulum bob. Pass the other end through split cork and clamp it in stand so that the bob hangs a little beyond the edge of the table. Let the point of suspension of pendulum *i.e.*, where it enters the split cork, be less than 20 cm above the table.

2. Mark a large protractor on the card board in units of 10°. Its centre may be at mid-point of the longer side. Right bisector of longer side is marked 0°, and angles upto 90° are marked on its left and right sides. Fix it on the edge of the table by two drawing pins so that its centre is close to the point of suspension of the pendulum, and 0° line coincides with the pendulum at rest.

3. Oscillate the pendulum with a small amplitude (A), say about $\frac{1}{4}$ th of 10°, in a plane parallel to the card board. Examine the stop watch/clock and find its least count. Also determine its zero error if any. Measure time taken by, say 20 oscillations. Start the watch when the pendulum crosses the vertical line, while moving, say, towards left. Start counting from zero

in order to reduce chances of an error in counting. Stop the watch at the count 20, when the pendulum again crosses the vertical line moving towards left. Repeat measurement at least thrice and take their mean. Compute the time for one oscillation *i.e.*, the time-period (t).

4. In a similar manner, measure time-period for oscillations with amplitude of 5°, 7·5°, 10°, 12·5°, 15°.

5. Plot a graph between t and A^2 taking independent variable A^2 along X-axis and t along Y-axis.

Because change in t may be quite small, the scale for t should not start from $t = 0$. It should start with a value close to the value of t for small amplitude and its scale should be highly enlarged. Due to enlarged scale for t, the points plotted may not lie on the graph and one or two may be rather widely away from it. Draw a straight line as close to all points as possible.

Observations and Calculations :

Least count of stop-watch/clock=.........s
Zero error of stop-watch/clock =.........s

S. No.	*Amplitude (A)*	*Corrected time for 20 Oscillations in s*				*Time Period* $t = \frac{T}{20}$ *in sec*	A^2
		1	2	3	*Mean (T)*		
1.							
2.							
3.							
4.							
5.							
6.							

Result :

(*i*) Least count of stop-watch/clock=...........*s*

(*ii*) The graph between *t* and A^2 is a straight line. This means that the time period of the pendulum remains the same although the amplitude goes on varying. In other words the time period of a simple pendulum is *independent of amplitude*.

Precautions and Sources of Error

Same as in Expt.16.

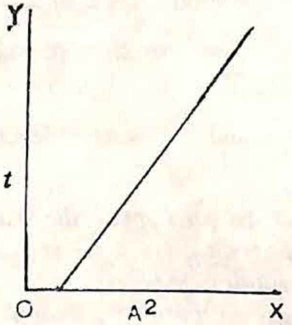


Fig. 6·2

SECTION B

Experiment 7 :

To study the rate of flow of water filled in a burette (or a vertical cylindrical container) connected to a capillary and to find its half-life.

Or

To find the time required to empty a burette full of water to $\frac{1}{2}$ of its volume; $\frac{1}{4}$ of its volume; $\frac{1}{8}$ of its volume and so on and to draw a graph between

(i) volume of water emptied (V) and time (t).

(ii) volume of water left and time and to study if at every stage the rate of flow is the same.

(iii) log V and t. What is the difference between these graphs ? Explain.

Apparatus :

Burette; funnel; clamp stand; graduated jar or breaker; stop-watch; capillary tube (20-30 cm long); rubber tube and clip

Theory :

The rate flow of liquid is proportional to the hydrostatic pressure of water.

or $\frac{dV}{dt} \propto -P$ where P = hydrostatic pressure

and $\frac{dV}{dt}$ = Rate of flow i.e., the volume of the liquid that comes out in one second.

Since, volume (V) = Ah

where A = area of cross-section and h is height of water column.

and $P = h\rho g$

where ρ = density of the liquid

and g = acceleration due to gravity.

$$\therefore \quad \frac{d(Ah)}{dt} \propto -h\rho g$$

$$\frac{dh}{dt} \propto -\frac{h\rho g}{A}$$

$$\frac{dh}{dt} \propto -\lambda h$$

where $\lambda = \frac{\rho g}{A}$ = decay constant. or $\frac{dh}{h} \propto -\lambda dt$

Integrating the above equation we get,

$$\int_{h_0}^{h} \frac{dh}{h} = -\lambda \int_{0}^{t} dt$$

or $\log \frac{h}{h_0} = -\lambda t$ or $\boxed{h = h_0 e^{-\lambda t}}$ which is an exponential relation.

It is the decay constant λ which decides the quickness with which the rate of flow reduces because in the exponential fall, the rate of flow decrease much faster in the beginning but it reduces considerably as the time elapses.

Decay constant is the reciprocal of the time during which the volume of water in the burette becomes $\frac{1}{e}$ of its original volume. Decay constant depends upon the opening of the bore of the stopper. For wider bore λ is larger and vice versa. For larger values of λ the decrease in the rate of flow is faster and vice versa.

Procedure :

1. Clamp the burette exactly vertical and attach a *horizontal* capillary tube to the bottom of the burette by means of rubber tubing as shown in Fig. (7.1). Slip the clip (or pinch cock) over the rubber tubing.

2. Measure the height (h_0) of the zero mark from the lower end of the jet. This can be done by measuring the vertical height below the lowest graduated mark on the burette.

3. Fill the burette upto the zero mark by using a funnel.

4. Divide the height (h_0) in 2, 4, 6, 8, 16 or 32 parts and note the time water takes to empty upto the level keeping the *same flow*.

(Apart from these observations some more observations should be recorded corresponding to greater heights so as to obtain large number of points for the graph).

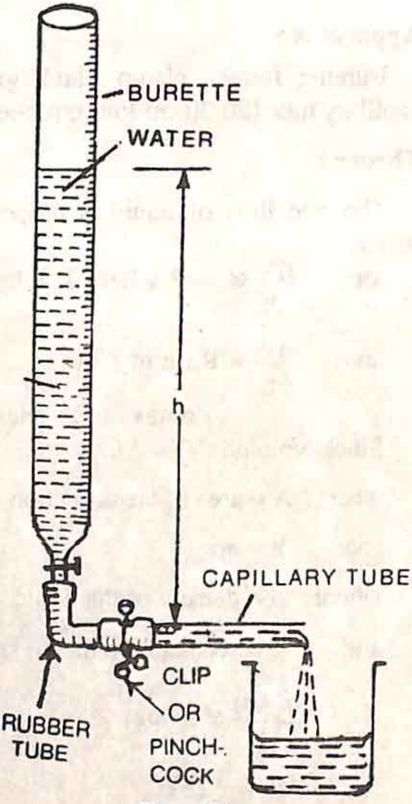


Fig. 7.1

5. Plot a graph between level of water in the burette (h) and time (t) and also between (h_0—h) and t which come out to be exponential curves.

6. Plot a graph between log h and t and also between log (h_0—h) and t and calculate λ using the relation

$$\lambda = \frac{\log h_0 - \log h}{t}$$

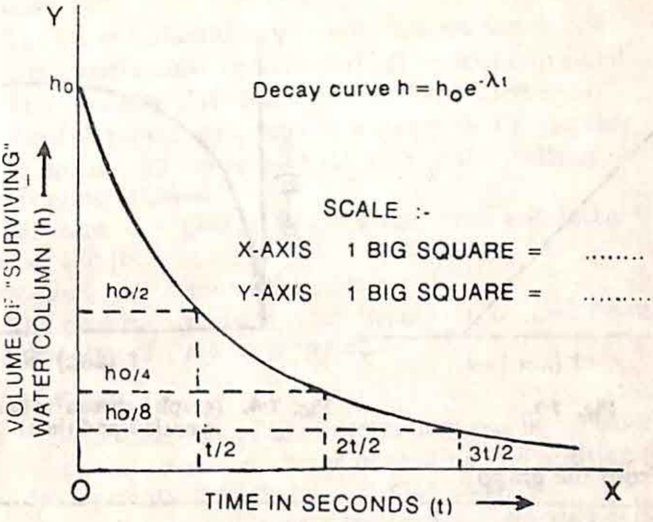


Fig. 7·2.

7. Observe the slope at various points in Fig. 7.2. It will be found that the slope is different at different places. In fact it will be seen that $\frac{dh}{dt}^{-}/h$ is constant.

Observations and Calculations :

Least count of the stop watch=......sec

Distance from the lowest mark of the burette to the tip of the jet=......

h_0=......

log h_0=......

No. of obs.	*Level of water in the burette (h)*	*Time in seconds at level h (t)*	*log h*	$\lambda=\frac{\log h_o-\log h}{t}$	(h_o-h)	$\log(h_o-h)$

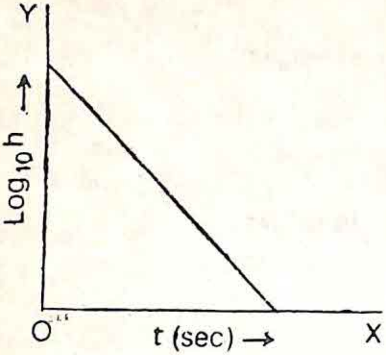


Fig. 7·3.

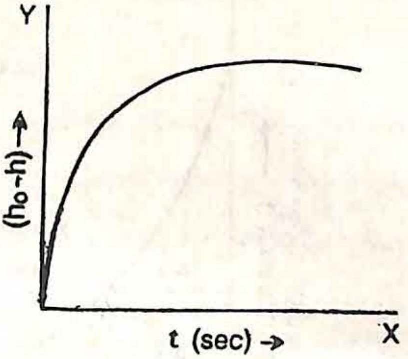


Fig. 7·4. (graph between volume emptied and time)

From the graph

S.N.	$\frac{dh}{dt}$	*h*	$\frac{dh}{dt}\Big/ h$ = *constant*	*Half life in sec.*
1.				
2.				
3.				
4.				

Result :

(*i*) The graphs between h and t and between (h_0-h) *i.e.*, volume emptied and t are *exponential curves* ***i.e.*** exponential decreasing curve and exponential increasing curve respectively.

(*ii*) The rate of flow at every stage is *different i.e.*, goes on decreasing exponentially but $\frac{\frac{dh}{dt}}{h}$ is found to be constant.

$$\left[\frac{\frac{dh}{dt}}{h}=\lambda=\frac{\rho g}{A}=\text{constt.}\right]$$

(*iii*) Half life=.........sec

Precautions :

1. The burette should be of uniform bore throughout.
2. The burette must be kept vertical with clamp stand.
3. The capillary glass tube must be kept horizontal.
4. Read the lower meniscus of water with a set square.
5. Water should not remain sticking to the walls of the burette when the reading is taken.
6. Measure the height of the water level accurately from the lower end of the jet.
7. Record zero error of the stop watch.
8. The position of the stopper should in no case be disturbed while the draining of water is going on.

Sources of Error

(*i*) The graduations of the burette may not be accurate.

(*ii*) There is always an error in reading the position of water level and the stop watch simultaneously.

(*iii*) There is always a time lag between starting of the stop-watch and opening of the stopper.

ORAL QUESTIONS

Q. 1. *What is the analogy to this experiment* ?

Ans. Radio-active decay exhibited by Radio.active substances and discharging of a capacitor through a resistor.

Q. 2. *What is Radio-active disintegration (or decay)* ?

Ans. Heavy elements like uranium, thorium, polonium, radium etc., spontaneously disintegrate giving out α, β and γ-radiations. In this way new atoms are formed. This phenomenon is called Radio-active decay or disintegration. It is a completely haphazard or random process.

Q. 3. *How do you measure activity of a substance* ?

Ans. By the number of atoms disintegrating per second.

Q. 4. *What are Laws of Radio-active Disintegration* ?

Ans. Laws of radio-active disintegration :

(*i*) Atoms of every radio-active substance are constantly breaking into fresh radio-active products with the emission of α, β and γ-rays.

(*ii*) The rate of breaking is not affected by external factors (temperature, pressure, chemical combination, etc.) but depends entirely on the *law of change i.e.*, the number of atoms breaking per second at any instant is proportional to the number present.

Q. 5. *Define half-life period* ?

Ans. The half-life period of a radio-active substance is the time in which half of the radio-active substance will disintegrate.

Q. 6. *What is an activity and how is it expressed?*

Ans. The rate of decay of a source *i.e.*, the number of disintegrations per second is called its activity. It is often expressed in **curies**.

Q. 7. *Define curie.*

Ans. One curie is the activity of a source which on average undergoes $3{\cdot}70 \times 10^{10}$ disintegrations/sec.

Q. 8. *What is the first direct evidence that the phenomenon of radio activity gives?*

Ans. That all atoms are almost certainly made from the same building bricks.

Q. 9. *Define rate of flow of a liquid.*

Ans. It is the volume of a liquid that flows across a certain area of cross-section of the tube in *one* second.

Q. 10. *Why is the curve between volume emptied and time exponential?*

Ans. Because $h = h_0 e^{-\lambda t}$ is an exponential relation.

Q. 11. *How is the pressure at the nozzle of the burette or jet end of the capillary tube related with the height of water column.*

Ans. Pressure at the nozzle$=P+h\rho g$ where P is the atmospheric pressure ; h is the height of water column and ρ is the density of water, 'g' is the acceleration due to gravity. As h decreases, pressure also decreases with time.

Q. 12. *How are the rate of emptying and the rate of flow of water related?*

Ans. Both are equal.

Q. 13. *Is the rate of flow constant at every instant?*

Ans. It decreases exponentially with time.

Q. 14. *How will you get the rate of flow of water at any stage of flow?*

Ans. It is obtained by getting the slope $\frac{d(h_0-h)}{dt}$ of the graph between (h_0-h) and t at that instant.

Q. 15. *What is the nature of graph between* $\log(h_0-h)$ *and time* t*?*

Ans. It is a straight line.

Q. 16. *Why should a capillary tube of suitable bore and bent at 90° be attached to the nozzle of the burrette?*

Ans. The flow of water through the capillary becomes slower. It is because the internal diameter of the capillary is smaller than that of the nozzle of the burrette.

Q. 17. *How does the decay constant remain the same for different sets of observations? Why?*

Ans. By opening the stopper full in every observation. It is because the rate of flow remains smaller due to much small bore of the capillary tube.

Experiment 8.

To draw the s-t graph and v-t graph of the motion of hand as one walks using ticker-timer.

Apparatus :

Ticker timer, paper tape, G-clamp, plug key.

Diagram :

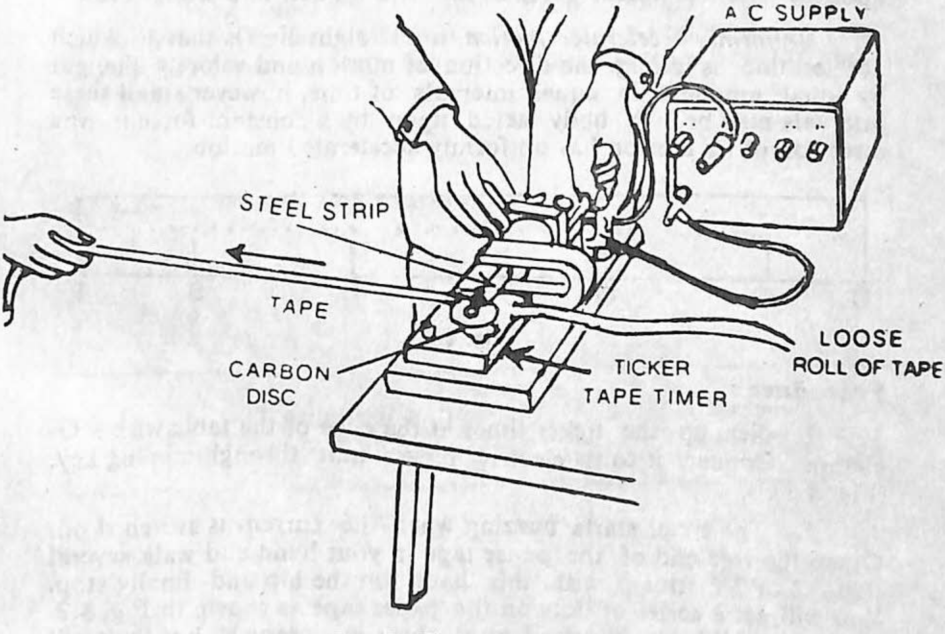


Fig. 8·1. Ticker tape timer

Theory : Ticker-Tape Timer (Vibrator and ticker-tape) :

The ticker-tape timer is a device used for measuring time intervals of the order of 1/50 or 1/100 of a second. Ticker tape timers of different designs are available. The one shown in Fig. (8·1) has a vibrating steel strip which strikes a small carbon disc under which a paper-tape is pulled. The strip is made to vibrate with the help of an electromagnet at a known frequency. When run on 6 volt stepdown A.C. supply, its frequency is the same as that of the A.C. mains *i.e.*, 50 hertz.

Another design is based on the electric bell, the gong being replaced by a mechanism of carbon disc and moving tape. Its frequency depends on mechanical properties of the hammer.

Marks (or dots) are obtained on the tape at equal intervals of time—*each dot marks a complete vibration of the strip. The time interval between two dots on the tape can be taken as the unit of time for some of the experiments*, which you may call a "**tick**".

Note :

The vibrator makes marks at equal *times* apart—and not necessarily at equal *distances* along the tape.

Uniform motion in a straight line is one in which a body covers equal distances in equal intervals of time, however small these intervals may be. A body not acted upon by any external force or acted upon by balanced forces executes uniform motion in a straight line.

Uniformly accelerated motion in a straight line is that in which acceleration is along the direction of motion and velocity changes by equal amounts in equal intervals of time, however small these intervals may be. A body acted upon by a constant force in the direction of its motion has uniformly accelerated motion.

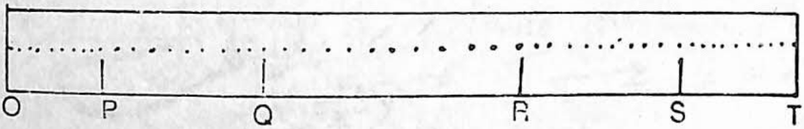


Fig. 8.2

Procedure :

1. Set up the ticker-timer at the edge of the table with a G-clamp. Connect it to its electric power unit through a plug key. (Fig. 8.1).

2. The timer starts buzzing when the current is switched on. Grasp the free end of the paper tape in your hand and walk several steps (2 or 3 metres), with this hand on the hip and finally stop. You will get a series of dots on the paper tape as shown in Fig. 8.2. The dots will have occurred at regular time intervals, but they will start *close* together, then be *farther apart*. Finally, the distance apart of dots will be very small.

3. The time interval between two consecutive dots on the paper tape will represent a unit of time, a "*tick*". What is the distance covered during any five ticks ? Is it constant ? By inspecting the tape you can find (*i*) where the velocity was highest and where it was lowest, (*ii*) where the velocity was increasing and where it was decreasing ?

4. Choose a distinct dot on the tape close to starting point as the origin (0) and mark every 10th mark conspicuous by another mark (P, Q, R.........) on it. Measure distance, *s*, of these marks from the origin and plot a *s*—*t* graph.

5. Also find the average velocity during the intervals OP, PQ, QR,......... These average velocities may be taken as the instantaneous velocities at the middle of the corresponding time interval. Suppose, for example, distance PQ on the tape is 20 cm. There are 9 dots marked on the tape by the timer between P and Q, *i.e.*, time interval of 10 ticks had elapsed. Then average velocity between P and Q is $\frac{20}{10}=2{\cdot}0$ cm per tick. It is roughly the actual velocity at an instant 5 ticks after the dot P was marked, *i.e.*, the instantaneous velocity at that time. In this manner compute data of instantaneous velocity versus time.

6. Now plot a v—t graph. From this graph you can find when the velocity was uniform. Also you can find from this graph, the regions (time intervals) during which the acceleration was highest and lowest.

Observations and Calculations :

For s—t graph

S. No.	*Time interval in "ticks"* (t)	*Distance* s (*in cm*)
1.		$OP=$
2.		$PQ=$
3.		$QR=$
4.		$RS=$
5.		⋮
6.		⋮
7.		⋮
8.		⋮

For V—t graph

S. No.	*Time interval in "ticks"* *(t)*	*Instantaneous velocity (v) in "cm per tick"*
1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		

Result/Remarks :

(*i*) From *s*—*t* graph :

(*ii*) From *v*—*t* graph :

Precautions :

1. The tape should remain taut during the experiment even if you happen to swing your hand slightly, If you swing your hand too widely it is likely that for a short duration, resultant motion of your hand is backwards. The tape will get loose during backward motion and will, therefore, not move. If this happens, reject this tape, and record a second walk.

2. The time interval between two dots on the tape can be taken as a unit of time which you may call a "tick". Where the measured time must be converted into the standard unit, second, the time period of the vibrating steel strip of the timer must be measured.

ORAL QUESTIONS

Q. 1. *Define motion of a body ?*

Ans. The continuous change of position of a body with respect to another is called motion.

Q. 2. *Is there anything like absolute motion ?*

Ans. No ; all motion is relative.

Q. 3. *What is displacement of a body ?*

Ans. A change of position of a body from one point to another is called its displacement. It is described in terms of two quantities, a *distance* and a *direction*. The displacement of a body will change if there is any change in either its magnitude or direction or both.

Q. 4. *Define uniform motion of a body.*

Ans. An object that moves through equal distances in equal intervals of time, however small these intervals may be, is said to be in uniform motion.

Q. 5. *What is non-uniform motion ?*

Ans. When a body moves over unequal distances in equal intervals of time, it possesses non-uniform motion.

Q. 6. *Distinguish between speed and velocity.*

Ans. *Speed* is the ratio of the distance travelled by any object, irrespective of its direction, to the time it takes to travel that distance.

The *velocity* of a moving body is the rate of change of position of the body in a particular direction. In other words, the velocity of a moving body may also be defined as its speed in a particular direction.

Both are measured in ms^{-1}.

Q. 7. *What is uniform velocity ?*

Ans. If a body not only traverses equal distances in equal intervals of time, however small these intervals may be, but also continues to move in the same direction, it is said to possess uniform velocity.

Q. 8. *Define instantaneous velocity of a body.*

Ans. The instantaneous velocity of a body is the actual value of the velocity of the body at any point of its path or at any instant of time.

Q. 9. *Define acceleration of a body.*

Ans. It is the rate of change of velocity of a body. Acceleration is the ratio of the change in velocity to the time over which this change occurs. Its SI unit is ms^{-2}.

Q. 10. *If the distance travelled by a body varies directly as the time, what conclusion do you draw about its motion and force ?*

Ans. The speed is *constant* and the force acting on it is *zero*.

Q. 11. *You observe an object covering distances in direct propotion to the square of time elapsed. What conclusion might you draw about its acceleration ? Is it increasing, decreasing, zero or constant ?*

Ans. The acceleration in this case is *constant*. If S is the distance covered and t, the time elapsed, then $S \propto t^2$

or $S=kt^2$ where k is a constant

We also know that $S=\frac{1}{2}at^2$ where a is a uniform acceleration.

$\therefore \quad k=\frac{1}{2}a$

i.e., a is constant.

Q. 12. *When does a body possess uniformly acceleration motion* ?

Ans. A body is said to move with uniformly accelerated motion if its velocity increases gradually *i.e.*, increases by equal amounts in equal intervals of time, however small these intervals may be *e.g.*, a body falling freely under the action of gravity ; a steel ball rolling down an inclined plane etc.

Q. 13. *What was Galileo's best known contribution to the science of kinematics* ?

Ans. Galileo's best known contribution to the science of kinematics was his discovery that the free fall acceleration is exactly the same for any object. Given an absence of air resistance, each object will fall to earth with exactly the same acceleration *i.e.*, 9·8 ms^{-2}.

Q. 14. *State Newton's first law of motion or law of inertia.*

Ans. According to Newton's first law of motion, every body continues to be in its state of rest or of uniform motion in a straight line unless and until it is compelled by some external force to change its state of rest or of uniform motion.

Q. 15. *State Newton's second law of motion.*

Ans. According to Newton's second law of motion, the rate of change of momentum produced in a body is directly proportional to the impressed force and takes place in the direction of the force applied.

Q. 16. *What do these laws give us* ?

Ans. Newton's first law of motion gives us the definition of force whereas Newton's second law of motion gives the measurement of force *i.e.*, $F=ma$ where F is the force applied on a mass m and a is the acceleration produced in it.

Q. 17. *Define force and give its SI unit.*

Ans. Force is that agency which produces or tends to produce, destroys or tends to destroy motion in a body.

It is measured by the product of mass and acceleration. Its SI unit is *newton* (N).

Q. 18. *How is one newton related to dyne* ?

Ans. 1 newton$=10^5$ dynes.

Q. 19. *Define energy. Give its SI and c g.s. units.*

Ans. It is the capacity of a body to do work. SI unit of energy is *joule* (J) and c.g.s. unit of energy is *erg*.

Q. 20. *How joule and erg are related to each other* ?

Ans. 1 joule$=10^7$ ergs.

Q. 21. *Name the two kinds of energy.*

Ans. Energy is of two kinds :

(*i*) Potential energy (P.E.)

(*ii*) Kinetic energy K.E.)

Q. 22. *Define P.E. and mention its SI unit.*

Ans. Potential energy is the energy possessed by a body due to its position or configuration. P.E. of a body$=mgh$ where m is the mass of the body, g is acceleration due to gravity and h is the height of the body from its zero level or mean position.

Its SI unit is *joule* (J).

Q. 23. *Define K.E. of a body and mention its SI unit.*

Ans. Kinetic energy of a body is the energy possessed by it due to its motion.

K.E.$=\frac{1}{2}mv^2$ where m is the mass of a body and v is its velocity.

Its S.I. unit is *joule* (J).

Q. 24. *State law of conservation of energy.*

Ans. According to law of conservation of energy ·

(*a*) energy can neither be created nor destroyed ;

(*b*) the total amount of energy in this universe is constant ;

and (*c*) When a certain amount of energy in one form is expended, its exact equivalent amount reappears in one or more of the other forms.

Q. 25. *What is the ultimate form of energy in all the interchanges in the form of energy ?*

Ans. Sooner or later, it assumes the form of *heat*, which in due course becomes distributed throughout space. It probably becomes unavailable, but is still there, though scattered

Q. 26. *Enumerate two examples of law of conservation of energy.*

Ans. (1) A body falling freely under gravity.

(2) A vibrating pendulum.

Q. 27. *What is dissipation of energy ?*

Ans. The energy which disappears as heat and cannot be used again for any useful purpose is said to have been dissipated.

Q. 28. *What is transformation of energy ?*

Ans. It is the change of energy from one form to another. For example when a body falls freely under gravity from a certain height, its P.E. changes into K.E. when it hits the ground.

Experiment 9 :

To study the changes in velocity of a body pulled by a constant force and measure its acceleration.

Apparatus :

Ticker-timer, paper tape, a horizontal laboratory table, a trolley, a bumper, 3 cm thick strip of wood, three G-clamps, pulley fixed on the bumper, hanger, slotted weights (100 *g* each or 50 *g* each), few bricks, plug key and a spring balance.

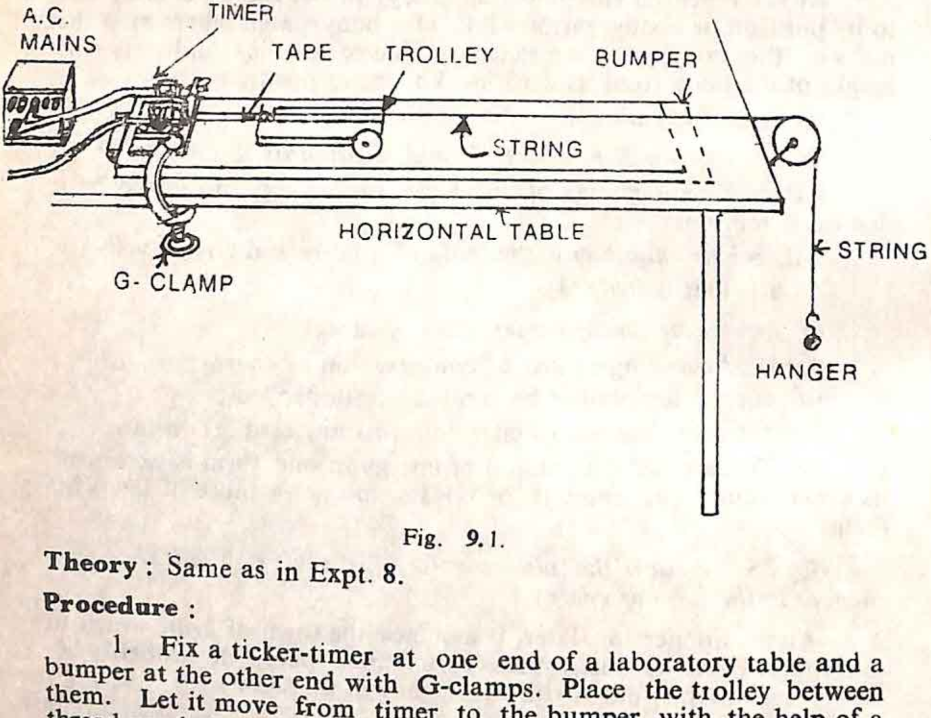


Fig. 9.1.

Theory : Same as in Expt. 8.

Procedure :

1. Fix a ticker-timer at one end of a laboratory table and a bumper at the other end with G-clamps. Place the trolley between them. Let it move from timer to the bumper, with the help of a thread passing over a *frictionless* pulley, at the other end of which is attached a hanger. *The run way is compensated for friction in the usual way.* Care must be taken that there is no drag by the ticker-tape comparable with the pulling load. In fact, the friction compensated should be arranged with the ticker tape in use.

2. Put slotted weights in the hanger. As it moves, the trolley pulls a length of tape through the ticker-timer. The mass of the pulley load must be only a very small fraction of the mass of the trolley.

3. Keep the trolley initially at rest near the timer, with hanger in its highest position. After switching on the timer, release the trolley. If the trolley runs very fast keep one or two bricks over it. The trolley gains speed till the hanger reaches the ground and thereafter it is stopped by the bumper. Make a mark on the tape at the point which was under the vibrator of the timer, when hanger touches the ground. Marks made by the timer upto this

point only are *useful as the force ceases to act after the hanger with weights touches the ground*. After the falling weight hits the ground, the trolley moves with a *constant velocity only*.

4. Choose a distinct mark on the tape close to starting point as the origin. Divide the entire motion in about 10 equal intervals of time. This can be done by counting equal number of dots, say 10, on the tape and marking the end-dot of each interval prominently.

5. Find the average velocity during each time interval $\left(v=\frac{s}{t}\right)$ which is quite close to the instantaneous velocity at the middle of each time interval.

More details in this regard may be seen in Experiment 8.

6. Plot a velocity versus time graph of the motion, taking a "tick" of the timer as the unit of time. Draw a smooth curve which best fits the points marked on the graph paper.

Find the slope at 5 or 6 different points along the entire curve. Find the average acceleration and estimate roughly the variation in it.

Observations and Calculations :

Constant force applied $=mg=$......N

S. No.	*Time interval in "ticks"* (t)	*Distance* s *in cm*	*Instantaneous velocity* $v=\frac{s}{t}$ *in "cm per tick"*
1.			
2.			
3.			
4.			
5.			
6.			
7.			
8.			

Slope of the Curve or Acceleration of the Body :

(*i*) Slope or acceleration $= \frac{AA'}{OA} = \ldots\ldots \text{cm/tick}^2$

(*ii*) Slope$= \frac{BB'}{OB} = \ldots\ldots \text{cm/tick}^2$

(*iii*) Slope$= \frac{CC'}{OC} = \ldots\ldots \text{cm/tick}^2$

(*iv*) Slope$= \frac{DD'}{OD} = \ldots\ldots \text{cm/tick}^2$

(*v*) Slope$= \frac{EE'}{OE} = \ldots\ldots \text{cm/tick}^2$

$\therefore$ Average acceleration$= \ldots\ldots \text{cm/tick}^2$.

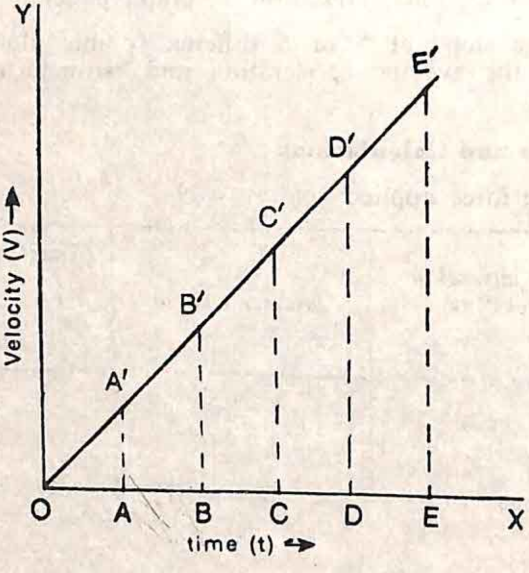


Fig. 9.2.

Result :

(*i*) The velocity versus time graph of the motion of a body pulled by a constant force is a straight line. The slopes of $v-t$ graph at various points are not different.

(*ii*) The average acceleration of the body when pulled by a constant force$= \ldots\ldots \text{cm/tick}^2$.

Precautions and Sources of Error :

1. Table must be clean, smooth and plane. The runway must be *compensated for friction.*

2. In case trolley has to be made heavier by placing bricks, crumbly bricks may be wrapped in paper to prevent their grit from falling on the table and thus obstructing the smooth motion of the trolley.

3. If the slopes of *v—t* graph at various points are different *i.e.*, if the graph is not quite a straight line, the following is most likely the reason for variation in the acceleration of the trolley (*i*) air resistance, (*ii*) different friction at different spots, (*iii*) an uneven top table, and (*iv*) a constant force may produce different accelerations at different instants of time.

ORAL QUESTIONS

(Same as in Expt. 8 and Expt. 10).

Experiment 10:

(*a*) *To locate the position of no friction for the junction of three threads when it is in equilibrium under the action of three forces in the parallelogram law of vectors apparatus.*

(*b*) *To find the weight of the given body using the parallelogram law of vectors.*

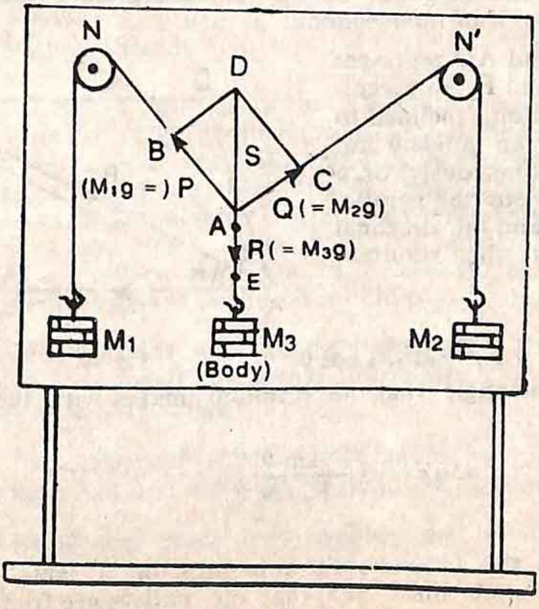


Fig. 10.1

Apparatus :

Gravesands' apparatus ; hangers ; slotted weights ; thread, half metre rod, plane mirror strip ; a spring balance, set squares and protractor ; a wooden block with a hook (unknown weight).

Theory :

If a number of forces act on a body simultaneously, it is possible to find a single force, which would replace them, *i.e.*, which would produce the same effect as is produced by all the given forces. This force is called the *resultant force*. If we apply to the body a force equal and opposite to the resultant, the body would be in equilibrium ; for the resultant and this applied force would balance each other. This applied force is, therefore, called the *equilibrant force*.

Since force is a vector quantity the resultant of two or more forces cannot be determined by applying the arithmetical laws, but by the method of compounding vectors. Thus, when we have only two forces acting on a body at a certain angle, we might compound them by the law of parallelogram of forces which may be stated as follows :

Law of Parallelogram of Forces

If two forces, acting simultaneously on a particle be represented in direction and magnitude by the two adjacent sides of a parallelogram, their resultant is represented in direction as well as magnitude by the diagonal of the parallelogram which passes through their point of intersection.

Let AB and AD represent two forces F_1 and F_2 in magnitude and direction, inclined to each other at an angle θ and impressed simultaneously on a particle. Complete the parallelogram ABCD and the diagonal AC represents the resultant force R.

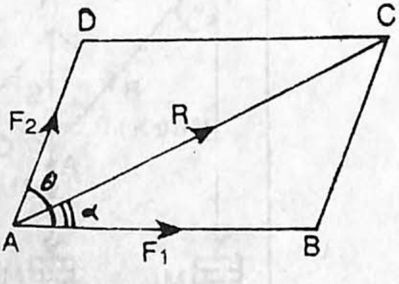


Fig. 10.2

$$R=\sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$$

If α be the angle that the resultant makes with the force F_1 then

$$\tan\alpha=\frac{F_2\sin\theta}{F_1+F_2\cos\theta}.$$

Procedure :

1. Place the Gravesand's apparatus on a table. Make it vertical by a plumb line. See that the pulleys are frictionless so as to rotate freely about their axis. *If not, oil them.*

2. Fix a sheet of paper on the board by means of drawing pins.

3. Pass a thread over the two pulleys with hangers at its ends and then knot another thread in its middle and attach the given wooden block (unknown weight) to its lower end. Add slotted weights to the hangers till the knot A (Fig. **10.**1) lies approximately at the centre of the sheet and none of the hangers and the given body touches the vertical board. When the weights are steady it would mean that the point A is in equilibrium under the action of three forces represented by the weights. In the case of perfect equilibrium, a slight disturbance of the point A should bring it back to the original position but generally the position of A may vary over a small area on account of friction at the pulleys, etc.

4. To mark the direction of the forces, place a plane mirror strip *lengthwise* under each thread in turn and mark two points one on either end by placing your eye in such a position that the image of the thread in the strip is covered by the thread itself.

5. Remove the hangers along with weights one by one and weigh them with a spring balance carefully and note the weights. Find the *zero error* of the spring balance before weighing.

6. Unclamp the board and place it on the table. Join the points marked under each string and produce them to meet at their common junction A which is the position of no friction for the junction of three threads.

7. Choose a suitable scale to indicate the forces so that you may get a fairly large parallelogram. From the common point A mark off lengths AB and AC representing the respective forces P and Q. On adjacent sides AB and AC complete the parallelogram ABCD and draw the diagonal AD which represents the resultant S of P and Q and mark arrow heads to indicate the direction of forces. Also produce EA (Fig. 10.1) and if the experiment is correctly performed, EA produced should pass through D.

8 Measure AD and calculate the force which it represents. It should be equal to its equilibrant in magnitude, *i.e.*, the unknown weight (R) of the given body.

9. Repeat the experiment twice more by changing weights in the hangers and find the mean value of the unknown weight R.

10. Verify the result by weighing the body with a spring balance.

Observations and Calculations :

Zero error of spring balance=......gm wt.

Scale : 1 cm=n (say) gm

No. of obs.	*Force*		*Diagonal AD (cm)*	*Resultant Force represented by the diagonal AD (S) in g* $=AD\times n$	*Unknown weight of the given body* $R=S=AD\times n$ *(g)*
	P	*Q*			
1.					
2.					
3.					

Mean value of unknown weight=.........g

Result :

(*a*) The common junction A is the position of no friction.

(*b*) The unknown weight of the given body by experiment=.........g

The unknown weight of the given body

∴ by spring balance=.........g

% error=.........

Precautions :

(*i*) The pulleys should be frictionless so as to rotate freely about their axes.

(*ii*) The apparatus should be vertical.

(*iii*) The weights should hang *freely* and not touch the board.

(*iv*) The direction of the forces should be marked with the help of a mirror strip placed *lengthwise* using a sharp pencil.

(*v*) The weights should be as large as possible to be so adjusted that the meeting point of the forces is near the middle of the paper.

(*vi*) Weights along with their hungers should be weighed with a spring balance, taking into account its *zero error* if any.

(*vii*) The scale should be so selected, as to get a fairly large parallelogram.

Sources of Error :

1. Weights may not be accurate.
2. Pulleys are not perfectly smooth.
3. Lines drawn may not be very sharp and fine and slight inaccuracy in making the directions.

ORAL QUESTIONS

Q. 1. *Define : Force.*

Ans. It is that agency which produces or tends to produce ; destroys or tends to destroy the state of rest or of uniform motion in a body.

Q. 2. *What is its unit in the SI system and define it ?*

Ans. *newton.*

One newton is that much force which produces in a mass of 1 kg an acceleration of 1 m s^{-2}.

Q. 3. *How kilogram weight is related to newton ?*

Ans. 1 kg weight=9·81 newtons

Q. 4. *Define weight of a body. What is its unit ?*

Ans. Weight is the pull exerted by the earth on the body and is equal to mass × acceleration due to gravity. It is obtained by a spring balance.

It has the same units as force, *i.e.*, newton and kg wt.

Q. 5. *What do you mean by resultant force and component forces ?*

Ans. *Resultant* force is that single force which produces the same effect on a body as is produced by a large number of independent forces. The various forces are called its *components*.

Q. 6. *What are scalar quantities ? How are they added up ?*

Ans. Those quantities which have *magnitude only* and no direction are called scalar quantities, *e.g.*, mass, length, time, speed, work, power, energy, etc.

They are added up *algebraically*.

Q. 7. *What are vector quantities ? How are they added up ?*

Ans. Vector quantities are those quantities which have magnitude as well as direction, *e.g.*, Force, Velocity, Displacement, Momentum, etc.

They are added up according to the Laws of composition of vectors, *i.e.*, by Parallelogram Law of vectors or Triangle Law of vectors or Polygon Law of vectors.

Q. 8. *What do you mean by composition of vectors ?*

Ans. It is a process of compounding two or more than two vectors into a single vector, *i.e.*, resultant vector.

Q. 9. *State Parallelogram Law of forces.*

Ans. *Parallelogram Law of forces* :

If two forces act on a particle simultaneously and they can be represented by the two adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through the point where the two sides meet.

Q. 10. *State Triangle Law of forces.*

Ans. *Triangle Law of forces* :

If two forces act on a particle simultaneously and they can be represented in magnitude and direction by the two sides of a triangle *taken in order*, their resultant is represented in magnitude and direction by the third side of the triangle taken in the *opposite order*.

Q. 11. *State Polygon Law of forces.*

Ans. *Polygon Law of forces* :

If a number of forces act on a particle simultaneously and they can be represented in magnitude and direction by the sides of a polygon taken in order, their resultant is completely represented in magnitude and direction by the closing side of the polygon taken in the opposite order.

Q. 12. *What is resolution of forces ?*

Ans. It is a process of resolving a resultant force into its component forces.

Q. 13. *What are rectangular components of a force ?*

Ans. If the components of a force are at right angles to each other, then they are called rectangular components.

Q. 14. *Distinguish between rest and equilibrium.*

Ans. *Rest* means motion and *equilibrium* means no acceleration. If a body does not changes its position with respect to its surroundings then it is said to be at rest and if a number of forces acting on a body simultaneously do not change its state of rest or of uniform motion then the body is said to be in a state of equilibrium.

Q. 15. *How did you find the unknown weight of a given body ?*

Ans. By applying Parallelogram Law of forces.

Q. 16. *Is it a good method ?*

Ans. No ; owing to the *friction* at the pulleys, it does not give accurate result.

[For further questions on friction read questions in Expt. 15]

SECTION C

Experiment 11 :

To find the downward force along the inclined plane acting on a trolley/roller on account of gravitational pull of earth and study its relationship with the angle of inclination of the inclined plane.

Apparatus :

Inclined plane with glass top with a pulley, pan, weight box, strong thread, roller or trolley, half metre scale, spirit level, spring balance.

Theory :

An inclined plane or a plane board hinged at its lower end, and supplied with some means of adjusting its inclination at several different values. At the upper edge of the inclined plane is usually fixed a pulley, over which a cord passes. To the end of the cord on the plane the load W is attached, while from the hanging end of the cord, weights from the weight box can be suspended to exert the force P on the cord (Fig. 11.1). In order to eliminate friction between the load W and the plane, W is usually a small roller, supported on an axle carried by a suitable framework, to which framework the cord is attached. In some forms of the apparatus the pulley and hanging weight are replaced by a spring balance which automatically adjusts itself to exert the required force P at any inclination, and P can be read off directly.

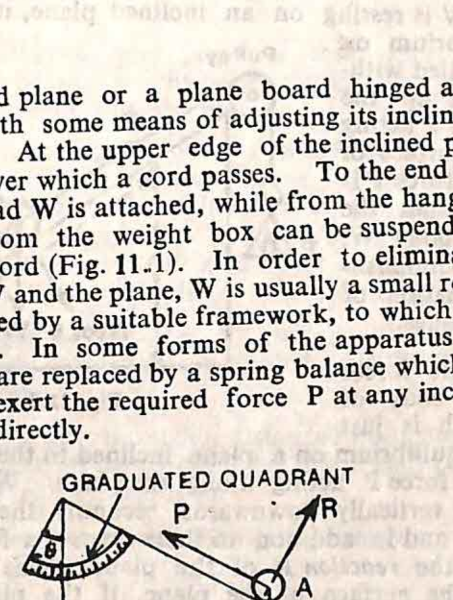


Fig. 11. 1.

The angle θ can be measured by means of a protractor placed with its edge along the horizontal base and its centre at the centre of the hinge but it is better to have a graduated Quadrant fitted near the top of the plane (Fig. 12.1), with a plumb line hanging from its centre. If the zero-line of the Quadrant stands out at right angles to the plane, the angle θ is read off as the angle between the plumb line and this zero line. This method is preferable to the first, as it is necessary to level the base by means of a spirit level if the first method is used, and other errors easily creep in when measuring the angle θ.

If a load W is resting on an inclined plane, it may be maintained in equilibrium on the plane, or pulled without acceleration up the plane, by a force P acting parallel to the surface of the plane. The force P is much smaller than the weight of the body W, the value of P diminishing as the inclination of the plane diminishes.

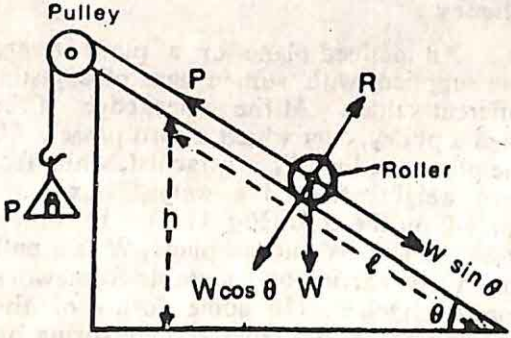


Fig. 11.2.

Consider the forces acting on a body of weight W, which is just maintained in equilibrium on a plane inclined to the horizontal, at an angle θ, by a force P acting along the plane. We have first its weight W acting vertically downwards, secondly the force P acting along the plane, and in addition to these there is a force exerted by the plane called the *reaction R* of the plane. This force acts perpendicular to the surface of the plane, if the plane be smooth (Fig. 11.2).

These three forces together keep the body at rest ; their directions are all known, and hence if the magnitude of one of them is known, the other two can be determined by applying the principle of the triangle of forces.

Let the line AB denote the weight W. Draw AC perpendicular to the plane, and parallel to the reaction R, and BC parallel to the plane, and parallel to P.

They intersect at C ; hence AC and BC represent R and P respectively.

The angle CAB is equal to the angle θ ; since AC is perpendicular to the plane and AB is perpendicular to the base. Thus

$$\frac{BC}{AB}=\sin\theta$$

But BC and AB represent P and W,

$$\text{therefore,}\quad \frac{P}{W}=\sin\theta$$

$$\boxed{P=W\sin\theta}$$

A somewhat simpler proof is obtained if we consider the *energy gained* by the weight when pulled up the plane, and *work done* by P in pulling it up the plane.

The work done by a force is measured by the product of the magnitude of the force and the *effective displacement*, or the distance moved by the point of application *in the direction of the force.* Suppose the force P pulls the body through a distance l, measured along the inclined plane, and in so doing raises it through a vertical distance h, which we may call the height of the plane.

The height of the top of the plane above the bottom is h, hence the weight gains an amount of potential energy$=Wh$, when raised to the top.

The force P acts through a distance l in its own direction when pulling the weight up the plane, if l is the length of the plane, hence the work done by the force is Pl.

By the principle of conservation of energy,

Energy gained = work done

or Output = Input

so that $Wh=Pl$

or $$\frac{W}{P}=\frac{l}{h}$$

Now Mechanical advantage of the Inclined Plane (M.A.)

$$=\frac{W}{P}=\frac{\text{Weight lifted}}{\text{Power applied}}$$

and velocity ratio (V.R.)$=\frac{l}{h}$

$$=\frac{\text{distance through which power is applied}}{\text{distance through which weight is lifted}}$$

$$\therefore \text{ Efficiency of the Inclined plane} = \frac{\text{M.A.}}{\text{V.R.}} \times 100\%$$

Also, $$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \times 100\%$$

Since $Wh = Pl$

$$\therefore \text{ Weight } (W) = P \cdot \frac{l}{h} = \frac{P}{\sin\theta}.$$

Thus knowing P and sin θ, the unknown weight (W) can be calculated.

Procedure :

1. First of all see that the pulley of the inclined plane and the roller rotate freely about their axis and if not, oil them.

2. Check up that the board with glass top is horizontal when it coincides with base board *i.e.*, when $\theta = 0$. If not, adjust it horizontal.

3. Suspend the spring balance in a clamp stand. Find the magnitude of each scale division and also find the zero error of the spring balance. Find the weight of the scale pan and that of the roller.

4. Pass a string over the pulley. Attach the roller lying on the plane to its one end and pan to the other end. Make sure that the thread does not touch any part of the horizontal board or the table.

5. Go on adding weights into the pan till the roller just begins to move upward on tapping the plane. Note the weights placed in the pan (p_1). Go on decreasing the weights placed in the pan now till the roller just begins to move downward on tapping the plane. Note again the weights placed in the pan (p_2). Take the mean of p_1 and p_2 *i.e.*, $\left(\frac{p_1+p_2}{2}\right)$ and add it to the weight of the scale pan to get the force applied (P) or the effort applied.

6. Apply different forces P over the pulley, using differen inclinations of the plane. Note corresponding values of P and θ, taking five or six different inclinations.

7. Plot a graph between sin θ and P and also between P and θ and compare the two graphs.

Observations and Calculations :

Zero error in the spring balance= ....g

Corrected weight of the scale pan= p= ....g

Corrected weight of the roller = W= ....g

S. No.	Weights placed in the pan when roller is moving		Component of gravitational force $(P)=p+\left(\frac{p_1+p_2}{2}\right)$ in gm	Inclination θ	$\sin\theta$
	up (p_1) in g	down (p_2) in g			
1.					
2.					
3.					
4.					
5.					
6.					

If the Quadrant for measuring θ is not fitted to the apparatus, measure h and l directly, taking *the same point on the plane each time* and measuring its height above the base and its distance *along the plane* from the centre of the hinge. The table of observations would then be arranged as follows :

S. No.	Weights placed in the scale pan when roller is moving		Component of gravitational force $P=p+\left(\frac{p_1+p_2}{2}\right)$ in g	h in cm	l in cm	$\sin\theta=\frac{h}{l}$	θ
	up (p_1) in g	down (p_2) in g					
1.							
2.							
3.							
4.							
5.							
6.							

Result :

(*i*) The downward force along the inclined plane acting on the roller/trolley on account of the gravitational pull of the earth

$$=P=\ldots\ldots\ldots g$$
$$=\ldots\ldots\ldots N$$

(*ii*) The graph between P and sin θ is a.........

The graph between P and θ is.........

Precautions :

(*i*) The pulley and the roller/trolley should be free from friction.

(*ii*) The thread between the roller and the pulley should be parallel to the plane.

(*iii*) The surface of the roller/trolley and that of the inclined plane should be clean, dry, plane and smooth.

Sources of Error :

1. The weights in the weight box may not be standard.
2. The pulley and the roller/trolley may not be frictionless.
3. The surface of the inclined plane may not be equally plane everywhere.

ORAL QUESTIONS

(Same as in Expt. 8 and Expt. 16).

Experiment 12.

To study how the acceleration of a body depends on the force acting on it and the mass of the body.

Apparatus :

Ticker tape timer, loose roll of tape, carbon disc, 4 trolleys, raised runway or a horizontal laboratory table, metre-rule, elastic cords with eyelets, a.c. supply.

Theory :

Newton's Second Law of Motion can be stated as "the rate of

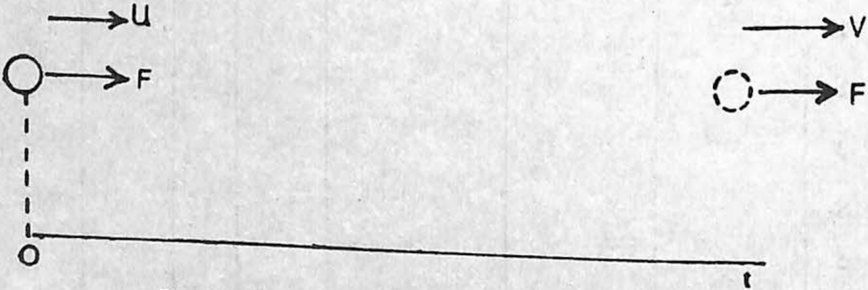


Fig. 12.1. A mass being uniformly accelerated.

change of momentum of an object is equal to the unbalanced force applied to it causing the change in momentum to take place in the direction of the force".

Consider a mass m moving with an initial velocity u. It is accelerated to a velocity v in a time t by the application of a constant unbalanced force F throughout the t seconds (Fig. 12.1).

Change in momentum

$$\Delta p = +mv - (+mu)$$

$$\Delta p = mv - mu$$

Rate of change of momentum

$$\frac{\Delta p}{t} = \frac{mv - mu}{t}$$

Unbalanced force $\mathrm{F} = \dfrac{m(v-u)}{t}$

$$\boxed{\mathrm{F} = ma}$$

where a=acceleration which by definition is the rate of change of velocity.

The Force F has units of kg $\mathrm{ms^{-2}}$ which are known as newton (N).

From Newton's Second Law expressed in the form $\mathrm{F} = ma$ it can be seen that, for a fixed mass m, the acceleration a is directly proportional to the applied unbalanced force F.

$$a \propto \mathrm{F}$$

The equation $\mathrm{F} = ma$ also states that for a given unbalanced force F, the acceleration a is inversely proportional to the mass m *i.e.,*

$$a \propto \frac{1}{m}$$

or $$a = \frac{\mathrm{F}}{m}$$

Procedure :

[A] To test the relationship a $\propto$ F :

1. Set up the apparatus shown in Fig. **12.** 2 but without the elastic cord attached.

The first requirement is to compensate for the frictional force between the trolley and the runway or the horizontal laboratory table. This is done by raising one end of the runway or horizontal laboratory table until a trolley, with a tape attached, shows uniform velocity (*i.e.*, the dots on the ticker tape are equally spaced) when it is given a gentle push down the inclined runway or horizontal laboratory table.

2. Now attach one elastic cord to the single pillar on the trolley and stretch it until it is level with the double pillars. A force F is required to stretch the cord, and this force F will remain constant provided the length of the stretched cord is kept constant while you pull the trolley down the runway or horizontal laboratory table. Thread part of a 3 m length of ticker tape under the carbon disc and then stick it on one end of the trolley.

Diagram :

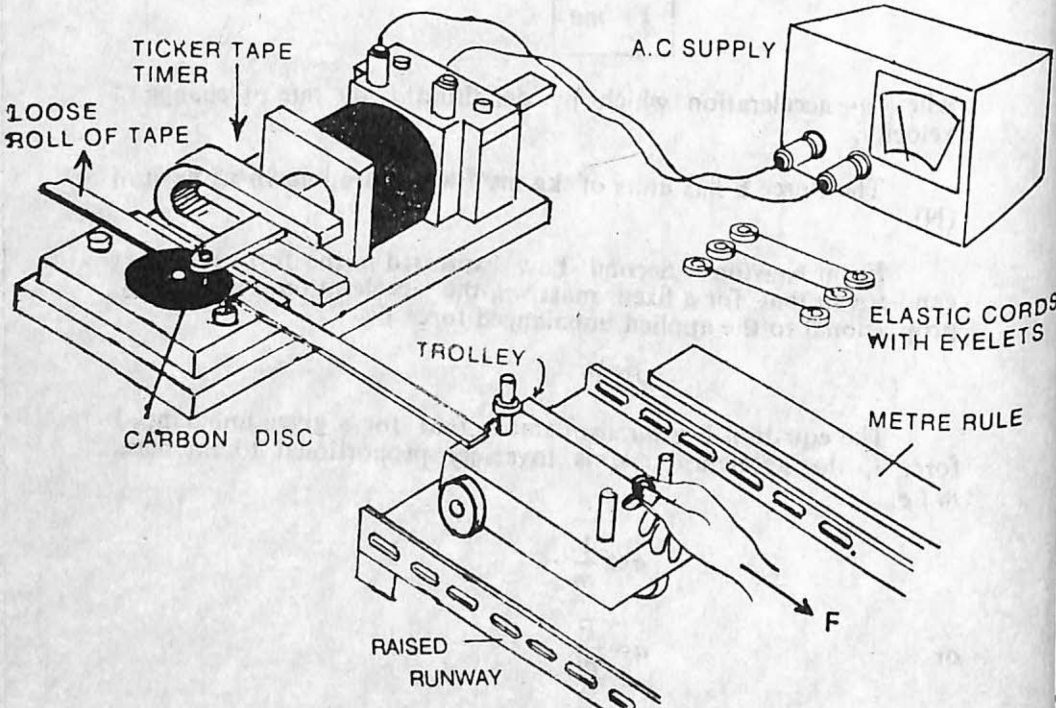
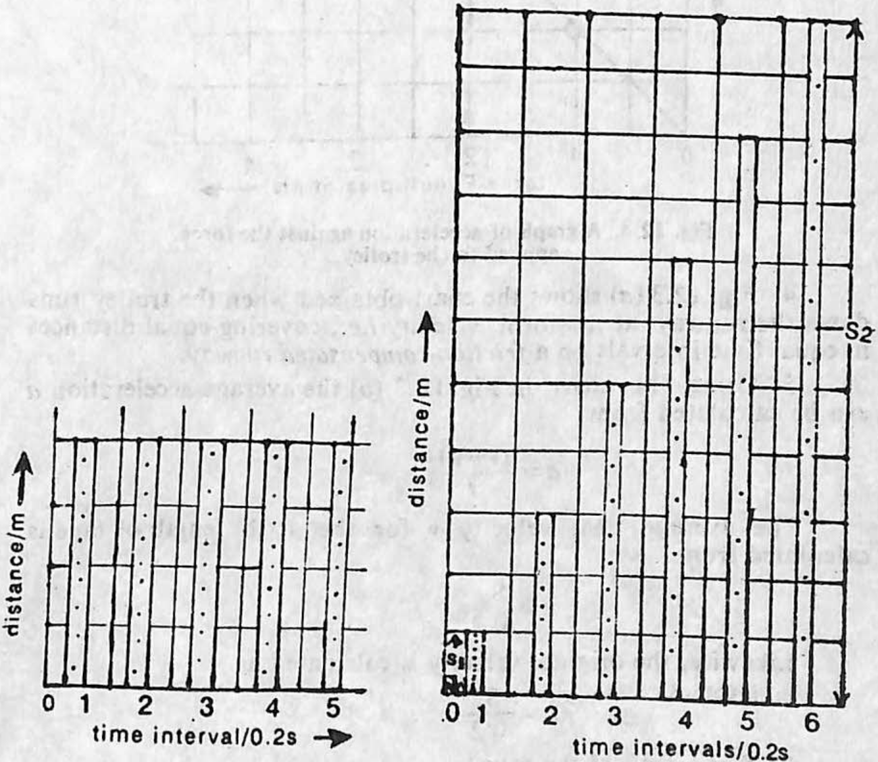


Fig. 12.2.

Start the timer and stretch the cord while still holding the trolley. After you release the trolley you must move alongside the runway holding the stretched cord so that its length remains constant. *You may have to practise this because the cord will try to shorten when the trolley is first released and you must apply just the right force to prevent this happening.*

3. After a successful run, detatch the tape and, disregarding the first series of dots, cut it up into strips which each represent 10 vibrations or 0·2 s (10×0·02 s). These strips can be stuck on a sheet of graph paper to give a chart like that in Fig. 12.3 (*b*).



(*a*) (*b*)

Fig. 12.3. Tape charts for (*a*) uniform motion on the friction-compensated runway and (*b*) accelerated motion.

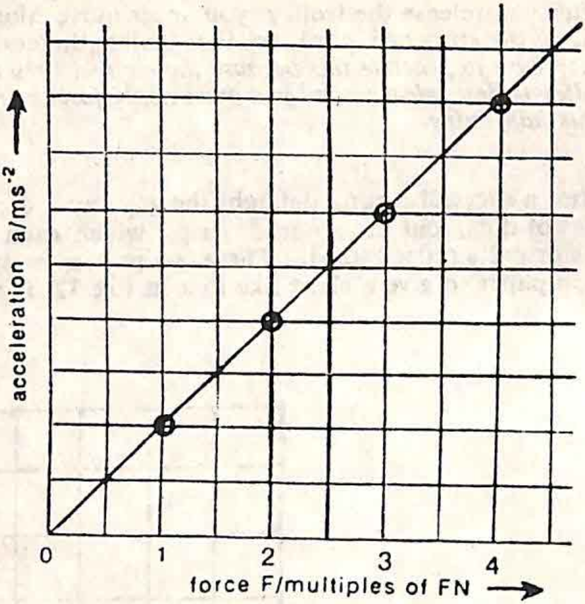


Fig. 12.4. A graph of acceleration against the force applied to the trolley.

4. Fig. 12.3 (*a*) shows the chart obtained when the trolley runs down the runway at uniform velocity *i.e.*, covering equal distances in equal time intervals on a *friction-compensated runway.*

5. Using the chart in Fig. 12.3 (*b*) the average acceleration *a* can be calculated from

$$a=\frac{(v-u)}{t}$$

The average final velocity v for the sixth length of tape is calculated from

$$v=\frac{S_2}{0{\cdot}2}$$

Likewise, the original velocity is calculated as

$$u=\frac{S_1}{0{\cdot}2}$$

from the first length of the tape.

The time interval t between the first and sixth tapes is $5\times0{\cdot}2$ s$=1{\cdot}0$ s. Hence

$$a=\frac{\frac{S_2}{0{\cdot}2}-\frac{S_1}{0{\cdot}2}}{1}=\frac{S_2-S_1}{0{\cdot}2}\ \text{ms}^{-2}$$

6. When two elastic cords of the same original lengths and made of the same elastic are both stretched by the same amount as the one in the first part of the experiment, the new force will be twice the original force.

Repeat the experiment using two, three and four stretched cords to apply forces of 2F, 3F and 4F respectively to the same trolley. In each case calculate the acceleration *a* as shown above.

7. Plot a graph of '*a*' against 'F'. It should be a straight line passing through the origin as shown in Fig. **12**.4.

[B] To test the relationship $a \propto \frac{1}{m}$:

(*i*) Again set up the apparatus shown in Fig. **12**.2 but this time keep the accelerating force F constant by using two or three elastic cords and instead vary the mass the force acts upon. First run a single trolley, then two, three and four trolleys stacked together (Fig. 1**2**.5.) *All the trolleys should be of equal mass m*, so that the total mass is increased by the same amount each time. Alternatively, you could add metal blocks or standard masses equal in mass to the trolley, to a single trolley.

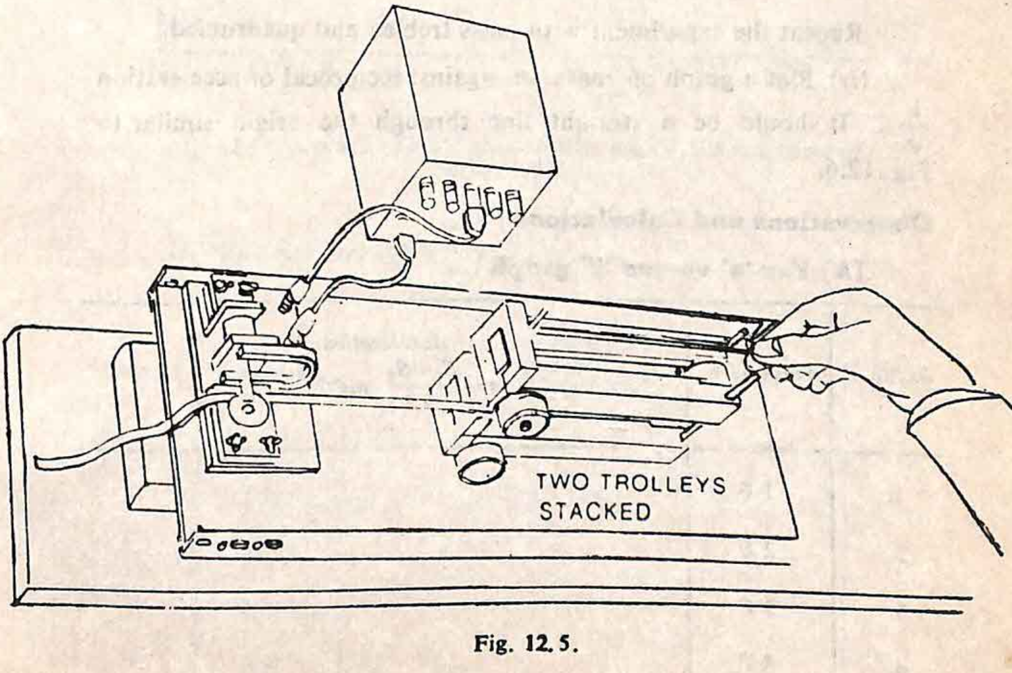


Fig. **12**.5.

Once again one end of the runway must be raised to compensate for friction so that the trolley with the ticker tape attached will move with uniform velocity (*i.e.*, the dots on the ticker tape are equally spaced) when given a gentle push down the inclined runway.

(*ii*) When the preliminary adjustments are complete, attach a 3 m length of tape to the trolley and apply a steady force using two or three stretched elastic cords. Using the technique shown in Fig. 12.3 (*b*), calculate the acceleration a and hence find $\frac{1}{a}$ for the single trolley.

(*iii*) Now double the mass to be accelerated by stacking a second identical trolley on the top of the first one or by adding standard masses equal in value to the mass of the trolley. Since the mass is now double, the weight is also doubled and therefore the frictional force will increase. Thus the runway must be raised a little higher so that it is friction-compensated for the double mass.

Now accelerate the double mass using the same two or three cords and determine a and $\frac{1}{a}$ from the tapes.

Repeat the experiment with mass trebled and quadrupled.

(*iv*) Plot a graph of mass m against reciprocal of acceleration $\frac{1}{a}$. It should be a straight line through the origin similar to Fig. 12.6.

Observations and Calculations :

[A] For 'a' versus 'F' graph :

S. No.	*Force (F)*	*Acceleration* $a = \frac{S_2 - S_1}{0{\cdot}2}\ ms^{-2}$
1.	1 F	
2.	2 F	
3.	3 F	
4.	4 F	

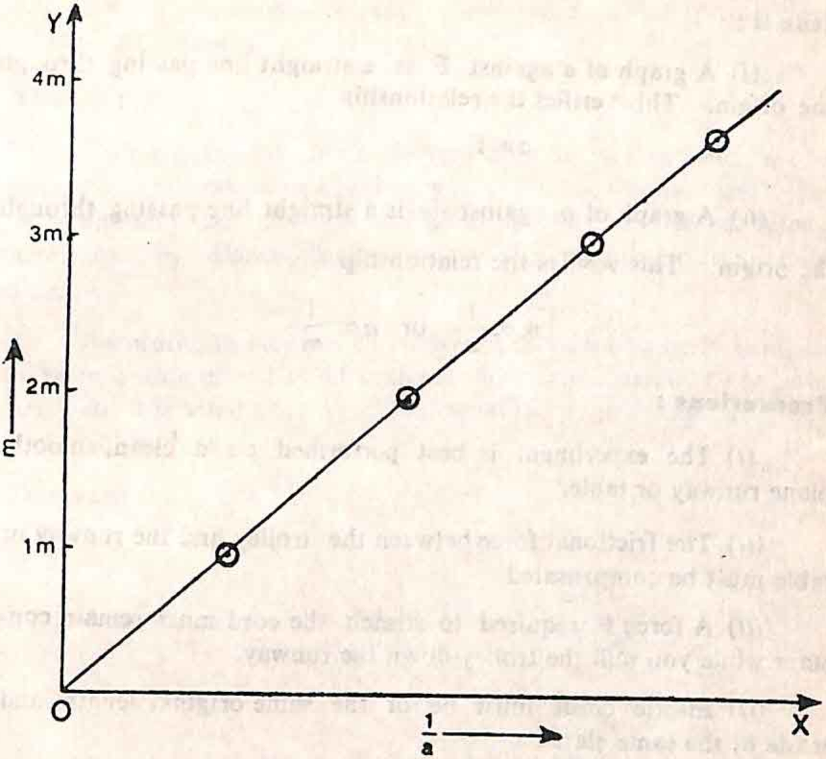


Fig. 12.6. A graph of mass of the trolley against the reciprocal of its acceleration.

[B] For 'm' versus $\frac{\text{‘1’}}{a}$ graph :

S. No.	Mass (m)	Acceleration $a = \frac{S_2 - S_1}{0.2}\ ms^{-1}$	$\frac{1}{a}$
1.	1 m		
2.	2 m		
3.	3 m		
4.	4 m		

Result :

(*i*) A graph of a against F is a straight line passing through the origin. This verifies the relationship

$$a \propto F$$

(*ii*) A graph of m against $\frac{1}{a}$ is a straight line passing through the origin. This verifies the relationship

$$m \propto \frac{1}{a} \quad \text{or} \quad a \propto \frac{1}{m}.$$

Precautions :

(*i*) The experiment is best performed on a clean, smooth, plane runway or table.

(*ii*) The frictional force between the trolley and the runway or table must be compensated.

(*iii*) A force F required to stretch the cord must remain constant while you pull the trolley down the runway.

(*iv*) Elastic cords must be of the same original lengths and made of the same elastic.

(*v*) All the trolleys should be of *equal* mass so that the total mass is increased by the same amount each time.

ORAL QUESTIONS

(Same as in Expt. 8).

Experiment 13.

To study the first law of motion by Galileo's experiment, using the double inclined track (i.e., a pair of inclined tracks each of which can be independently set at a small angle of inclination).

Apparatus :

A double inclined track (or a flexible curtain rail), a ball bearing of diameter 2·5 cm or more or a large marble, an accurate spirit level, about 15 to 20 small glass plates each of size about 3 cm

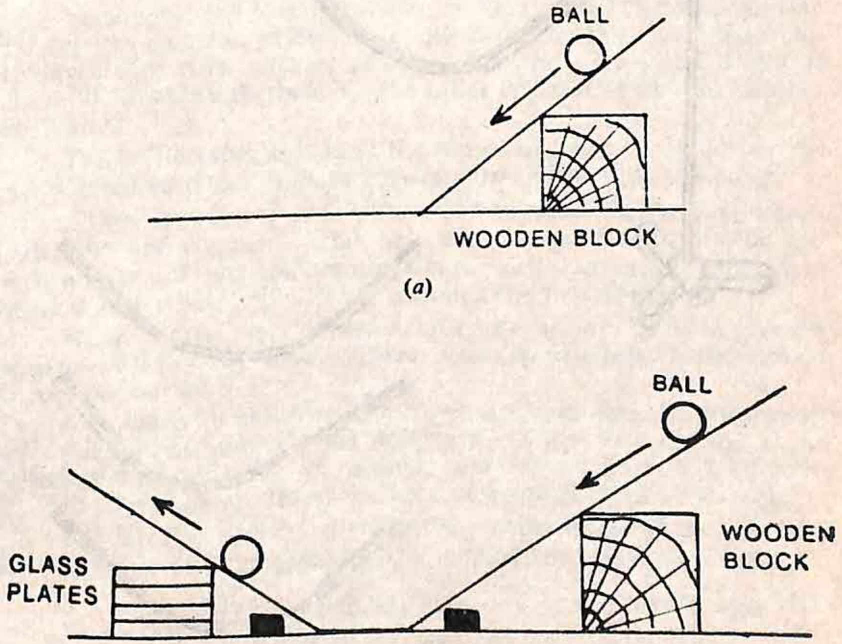
×3 cm×2 mm, wooden block, screw gauge ; 2 laboratory clamp stands ; 2 G-clamps.

Theory :

Uniform motion in a straight line is one in which a body covers equal distances in equal intervals of time, however small these intervals may be. A body not acted upon by an external force or acted upon by balanced forces executes uniform motion in a straight line.

According to Newton's first law of motion : "a body continues to be in a state of rest or of uniform motion in a straight line unless and until it is acted upon by an external force to change its state of rest or of uniform motion."

Diagram :



(*a*)

(*b*)

Fig. 13.1

Alternative Set up of the Apparatus :

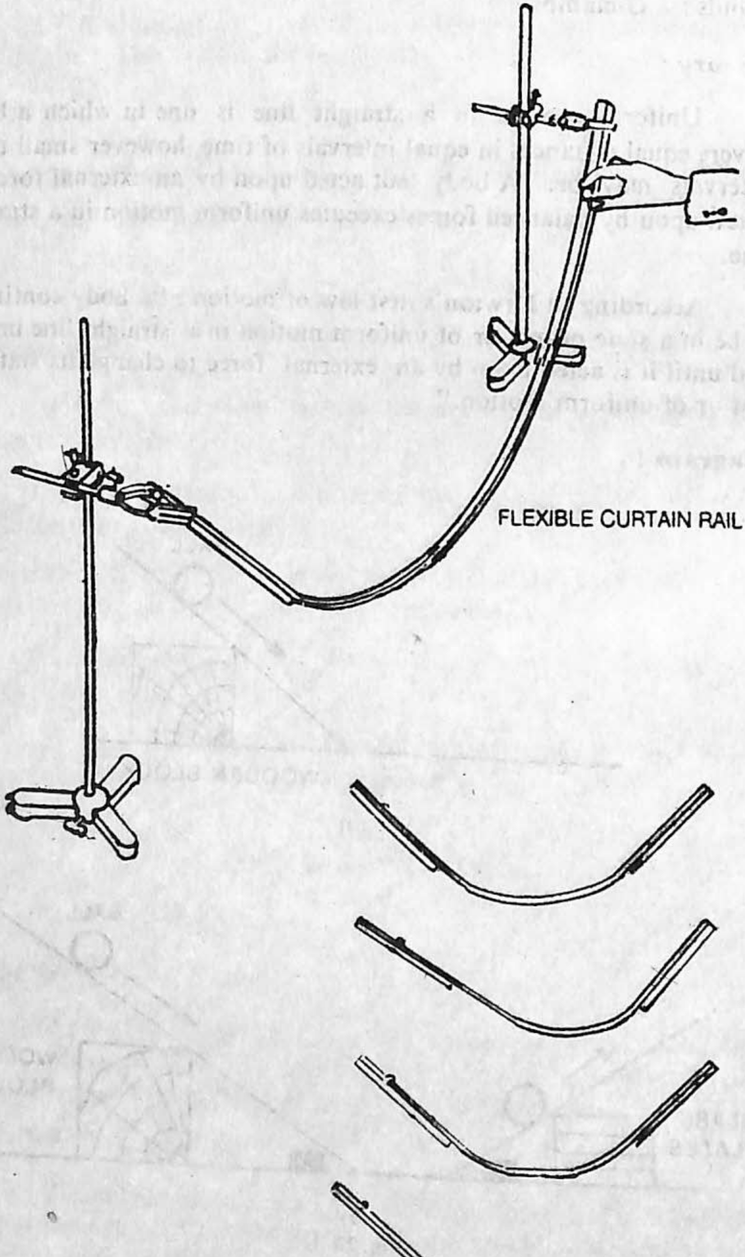


Fig. 13.2.

Procedure :

1. Lay the double inclined track on the table and clean it with cotton or tissue paper.

2. Hold the mid-point of the apparatus with the help of pair of weights. Insert a wooden block under the right arm such that it is raised by 3 to 4 cm. Make the other arm (say left) horizontal [Fig. **13**.1 (*a*)] using a spirit level. It can further be checked by placing a ball on this arm at different places. The ball should stay where it is kept. Each time we shall release the ball from the raised arm at a small fixed distance from the mid-point, say, at 5 cm or 10 cm.

3 Now raise the end of left arm using glass plates [Fig **13**.1 (*b*)] Measure the thickness of glass plates by using screw gauge.

It is advisable to check up by a screw gauge that the glass plates are of equal thickness. These should be cut from the same larger glass plate. If these are of different thickness, write the thickness on each with a spirit based pen. Then for each inclination of the left arm of the apparatus, calculate the height through which its end is raised above its horizontal position.

Alternative Set up of the Apparatus :

To support the curtain-rail is to glue a, 60 cm wooden lath (1·2 cm^2) to each end of the underside of the curtain-rail. One end is conveniently held with a retort stand and clamp at a height of about 30 cm above the bench. The other end can be held in another retort stand.

The ball bearing is held at the top of one end of the curtainrail and released so that it rolls down one side and then up the other.

As the curtain-rail is flexible it can be tilted, to various slopes, both equal and unequal. The ball can be released from each end in turn to see if any difference occurs. The experiment may also be tried with a horizontal length between the two slopes.

Note. *It is very important for the support to be rigid if the experiment is to be effective. There must not be energy losses caused by the rail moving.*

4. Release a ball from the right arm and note the distance *d*, upto which it ascends on the left arm. *Since the ball stays at the highest position only for an instant, you have to be alert.* Observe this distance of ascent three or four times and find the mean value.

Repeat the process for various inclinations of the left arm. The inclination can be changed by changing the number of plates.

5. Plot a graph between the slope of the left arm and $\frac{1}{d}$. Since the slope is proportional to the number of glass plates, you may simply plot $\frac{1}{d}$ versus number of glass plates. Extend the graph to slope zero *i e*., horizontal position of this arm.

Observations and Calculations :

S. No.	*Slope (No. of glass) plates*	*Distance of ascent (d) in cm*						$\frac{1}{d}$ *in* cm^{-1}
		1	*2*	*3*	*4*	*5*	*Mean*	

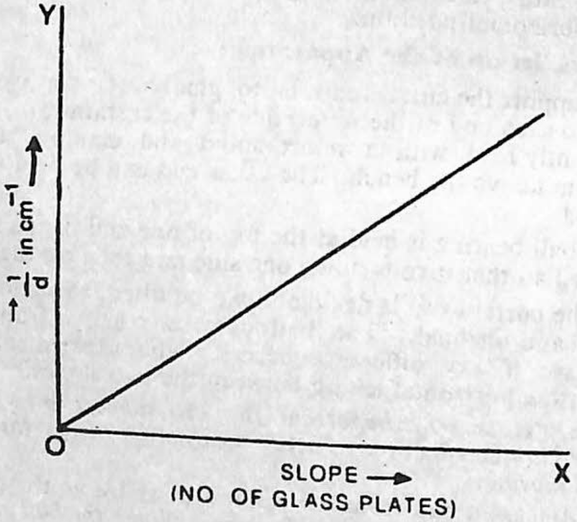


Fig. 13·3.

Result :

The graph between $\frac{1}{d}$ versus number of glass plates (*i.e.*, the slope) is a straight line. By extending the graph to zero slope, we find that the value of $\frac{1}{d}$ is zero for zero slope. It indicates that the ball will go on moving when the left arm is horizontal. This is what First Law of Motion says.

Precautions :

1. The double inclined track or the flexible curtain-rail must be properly cleaned. *Key to the success of this experiment is its low rolling friction. Even minute amount of dust or oil stain on the ball or on the track can cause much friction.* It is better to clean both by a tissue paper or cotton soaked in pure carbon tetrachloride.

2. Glass plates should be of equal thickness. These must be cut from the same larger glass plate.

3. Since the ball stays at the highest position only for an instant, one has to be alert to observe the distance of ascent.

4. The support must be rigid. There must not be energy losses caused by moving of the rail.

ORAL QUESTIONS

(Same as in Expt. 8).

Experiment 14.

(*a*) *To find the force of dynamic (sliding) friction for a wooden block on a horizontal plane.*

(*b*) *To find the force of rolling friction for a wooden block moving on rollers on a horizontal plane.*

Apparatus :

Wooden block with a hook, a pulley which can be fixed at the edge of a table and a flat plate of glass (or an inclined plane apparatus with glass top and pulley), a light scale pan, a weight box with fractional weights, a spring balance, spirit level and thread.

Diagram :

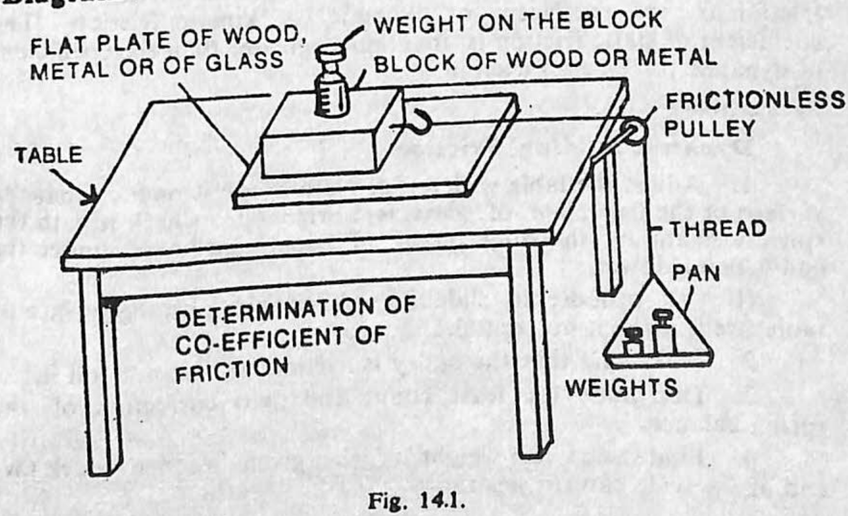


Fig. 14.1.

Theory :

The surfaces of bodies are never perfectly smooth. When two bodies are in contact and an attempt is made to slide one over the other, an opposing force called *force of friction* arises as a reaction to the applied force and acts in the opposite direction. The maximum force of friction that can be called into play between any two surfaces is known as *limiting friction* (F). This limiting friction is directly proportional to the *Normal reaction* (R) or the force acting normally on the surfaces in contact with each other.

Mathematically,

$$F \propto R$$

or $\boxed{F=\mu R}$ where μ is coefficient of friction.

This constant depends upon the nature of the surfaces in contact and the material of the bodies.

The force required to *start* two surfaces slipping over each other when pressed together with a given force is greater than the force necessary to keep them *moving* over each other when once they are started. We thus have two frictional forces for a given force perpendicular to the faces. One of these is called the **static frictional force,** being equal to the force that has to be exerted to start the motion, or **the force exerted on each other by the two surfaces when at rest**. The other force is called the **dynamic frictional force** being equal to **the force required to keep the surfaces in steady motion over each other when once the motion has commenced.** Corresponding to these two forces there will be two coefficients of friction. The coefficient of static friction and the coefficient of dynamic (or kinetic) friction. The coefficient of static friction is invariably greater than the coefficient of dynamic (or kinetic) friction.

Procedure :

Dynamic (sliding) Friction :

1. Adjust the table with a flat plate of glass on it so that the surface of the flat plate of glass is horizontal. Check it with the spirit level and by inserting pieces of paper/card board under the end which is lower.

(If you propose to slide the wooden block on the surface of table itself, make it horizontal.)

2. Make sure that the pulley is frictionless. If not, oil it.
3. Determine the least count and zero correction of the spring balance.
4. Find the correct weight of the given wooden block (W) and of the scale pan (P) separately.

5. Connect the hook of the wooden block with the help of a thread to the scale pan. Pass the thread over the frictionless pulley as shown in Fig. **14.**1 so that the scale pan hangs freely in the air. The portion of the thread between the hook and the pulley may be horizontal.

6. Tap the glass top gently and adjust the weight W in the scale pan so that the block moves *slowly* and *continuously* along the whole length of the glass plate. Note the total weight in the pan including the weight of the pan *i.e.*, P+*w*. In this situation the average force of dynamic friction on the glass top is equal to (P+*w*).

The motion of the block may have some jerks as the friction may be slightly greater at some portions of the glass top and less at others. Clean that portion by tissue or soft clean dry cloth and repeat the movement of the block.

7. Repeat the experiment at least six times by placing a weight of 20 g, 30 g, 40 g, 50 g, 100 g, 150 g etc , on the wooden block.

8. **Rolling Friction.** Next put *rollers e.g.*, round pencils, or glass rods slightly longer than the breadth of the glass top or nails with their heads cut off by a hacksaw etc., all along the glass surface and let the block move on them. Again find the force which moves the block slowly along the entire length of the glass top. In this case the light weight pan will have to be used and small weights (even milligrams weights) will be needed to achieve proper adjustment, because the rolling friction is very small.

In fact, rolling friction depends on the objects placed below the block to roll on. If time permits find the rolling friction for mustard seeds or glass marbles or small ball bearings etc., below the block to roll on.

Repeat the experiment at least six times by placing a weight of 20 g, 30 g, 40 g, 50 g, 100 g, etc., on the wooden block.

Observations and Calculations :

Least count of spring balance =........g

Zero error of the spring balance=........g

Weight of the wooden block

Observed=........g

Corrected=W=........g

Weight of the scale pan

Observed=........g

Corrected=P=........g

Dynamic (sliding) Friction :

S. No.	*Weight on the block (x) in g*	*Weight in the pan (w) in g*	*Force of Dynamic (sliding) friction (P+w) in N*
1.			
2.			
3.			
4.			
5.			
6.			
7.			

Rolling Friction :

S. No.	*Weight on the block (x) in g*	*Weight in the pan (w) in g*	*Force of rolling friction (P+w) in N*
1.			
2.			
3.			
4.			
5.			
6.			
7.			

Precautions :

1. The pulley should be free from friction.
2. The glass surface should be clean, bry and horizontal.

3. The thread between the hook and the pulley should be *parallel* to the glass surface.

4. The pan should not touch any part of the table.

5. The glass plate should be very gently tapped.

6. It may happen that during its jerky continuous motion, the wooden block may get stuck up at some spot and may not move on tapping the horizontal surface. This spot is not sufficiently clean. Clean that spot by tissue or soft clean dry cloth and repeat the movement of the block.

Sources of Error :

1. The weights in the weight box may not be standard.
2. The pulley may not be frictionless.
3. The horizontal surface of the glass plate may not be equally rough everywhere.

Note. The position of the wooden block on the friction table the direction of the wood fibres relative to it, and the moisture, all affect the magnitude of dynamic/rolling friction.

ORAL QUESTIONS

Q. 1. *What is friction ?*

Ans. When a body moves under the action of a certain force, the surface with which the body is in contact produces a counter force which resists the motion of the body by acting in the opposite direction. This resisting force is called friction.

Q. 2. *What are its causes ?*

Ans. It is due to the roughness of the two surfaces. The two irregularities of surfaces in contact interlock. Friction depends upon the force with which the bodies are pressed against each other.

Q. 3. *What is limiting friction ?*

Ans. It is the maximum force which is called into play when one body moves over the surface of another body.

Q. 4. *What is coefficient of friction ?*

Ans. It is the ratio of the limiting friction (F) to the normal reaction (R) for the two surfaces in contact, *i e.*, $\mu = \frac{F}{R}$.

Q. 5. *What are different kinds of friction ?*

Ans. Static friction ; sliding friction and rolling friction.

Q. 6. *Which is least of all ?*

Ans. Rolling friction.

Q. 7. *How will you reduce friction in your pulleys ?*

Ans. It is reduced by ball bearings where in a rolling contact is substituted for a sliding contact as rolling friction is always less than the sliding friction and by lubrication.

Q. 8. *Is friction of any use in nature ?*

Ans. Yes ; walking should have been impossible without friction and skating all the more difficult. Nails and screws would not hold nor would the fibres of a rope hold together etc.

Q. 9. *Why friction is an evil ?*

Ans. Because work done by a machine, *i.e.*, output is always less than work done on it, *i.e.*, input and hence the efficiency of a machine is always less than 100%. This is because a part of the energy supplied to the machine is used up in over-coming friction between the various parts of the machine.

Q. 10. *How will you reduce friction ?*

Ans. By making the two surfaces smooth. By using ball bearings arrangement and by shaping the body in a stream line by using lubricants, etc., etc.

Q. 11. *What is static friction ?*

Ans. When a body is made to slide over the other, a force of friction comes into play in the opposite direction. As the applied force is increased, the opposing force also increases. This opposing force of friction is called static friction so long as the body does not move.

Q. 12. *Is the static friction constant ?*

Ans. No. It increases with the applied force so long as the body does not move. It is a *self-adjusting* force.

Q. 13. *What is normal reaction ?*

Ans. When a body is placed over the surface of another body, the former gives action to the latter and the latter gives reaction to the former in the opposite direction. The magnitude of the reaction in the direction normal to the surface of the latter is called normal reaction.

Q. 14. *What are the laws of limiting friction ?*

Ans. (*i*) The magnitude of limiting friction between two surfaces is directly proportional to the normal reaction.

(*ii*) The magnitude of limiting friction is independent of the shape and area of the surfaces in contact so long as the normal reaction remains the same.

Q. 15. *Is the value of coefficient of friction same for all pairs of surfaces ?*

Ans. No ; the coefficient of friction is different for different pairs of surfaces.

Q. 16. *On what factors does the coefficient of friction depend ?*

Ans. It depends upon the nature and state of polish of the two surfaces in contact.

Q. 17. *What is dynamic friction or kinetic friction ?*

Ans. It is that force of friction which comes into play after the motion has started.

Q. 18. *What is rolling friction ?*

Ans. When a body rolls or tends to roll over the surface of another body, the resisting force called into play is known as the rolling friction.

Q. 19. *Which is greater ; static friction ; dynamic friction ; rolling friction ?*

Ans. Static friction is slightly greater than the dynamic friction whereas static friction and dynamic friction are very much greater than the rolling friction.

Q. 20. *How does rolling friction depend on the radius of the wheel ?*

Ans. The force of rolling friction varies inversely as the radius of the wheel.

Q. 21. *How does rolling friction depend upon rigidity of the two surfaces.*

Ans. It decreases as the rigidity of the two surfaces increases.

Q. 22. *Why do we not get the same value of coefficient of friction for different observations ?*

Ans. Because the surface over which a body moves is not uniformly polished and so the friction is different at different points; so the value of coefficient of friction also differs.

Q. 23 *What is angle of friction ?*

Ans. The angle which the resultant of normal reaction and force of limiting friction makes with the normal reaction is called angle of friction.

Q. 24. *What is the relation between angle of friction and coefficient of friction ?*

Ans. The tangent of the angle of friction is equal to the coefficient of friction, *i.e.*,

$\mu = \tan\theta$ where μ=coefficient of friction
θ=angle of friction.

Q. 25. *Why do we oil the pulley ?*

Ans. The pulley is oiled so that the friction at the bearing is minimised and the pulley is free to rotate.

Q. 26. *What is the disadvantage if the pulley is not frictionless ?*

Ans. A part of the applied force is utilized in overcoming the friction of the pulley and thus the observed value of the coefficient of friction will be greater than the actual value.

Q. 27. *Why do we slip on a rainy day ?*

Ans. On a rainy day, water forms a thin layer between the shoe and the ground and the friction becomes very much less. Thus a person slips.

Q. 28. *Why should the thread from hook to pulley be horizontal ?*

Ans. If the thread is slightly raised, the surface of the body to which thread is attached may be raised and thus the surface may not be in contact with the lower surface.

Experiment 15 :

To study the relationship between force of dynamic friction and normal reaction, between a block and a horizontal surface by drawing a graph. Can this be identified with some physical quantity ?

Apparatus :

A table with a frictionless pulley fixed at one end (or an inclined plane apparatus with glass top and pulley), a flat plate of glass, of brass or of zinc, blocks of different materials with hooks, a scale pan, a weight box, spirit level, spring balance a set of 5×200 g slotted weights, thread.

Theory :

Refer Fig. 14.1 and theory in Experiment 14.

Procedure :

1. Set the apparatus as in Experiment 15 and shown in Fig. 14.1.

Follow steps 1 to 6 in the procedure of Experiment 14 for dynamic friction.

Find the force of dynamic friction $(P+w)$ first with the wooden block alone sliding on the horizontal glass surface.

Also find the corrected weight w of the block with the help of spring balance, which is equal to normal reaction (R) between the wooden block and the glass top.

2. Next place one slotted weight (200 g) on the wooden block and measure the force of dynamic friction as well as normal reaction.

In this manner make these two measurements with various weights placed on the wooden block.

3. Plot a graph taking normal reaction (R) along X-axis and the dynamic friction $(F)=P+w$ along Y-axis. The graph will be a straight line upto a certain limit (Fig. 15.1).

(Draw a straight line with a ruler which best fits the points marked on the graph paper to represent your experimental data because measurement of force of dynamic friction is not a precise measurement and your data may not be able to distinguish between a straight line graph and a curved graph).

4. By covering the table with a flat plate of brass or of zinc and using blocks of different materials, the force of dynamic friction between several different pairs of surfaces can be determined.

Notes. You may have to make a judicious choice of the size of the block and the set of slotted weights for this experiment. If the block is too light, its force of dynamic friction may even be less than the weight of empty scale pan and you may not be able to

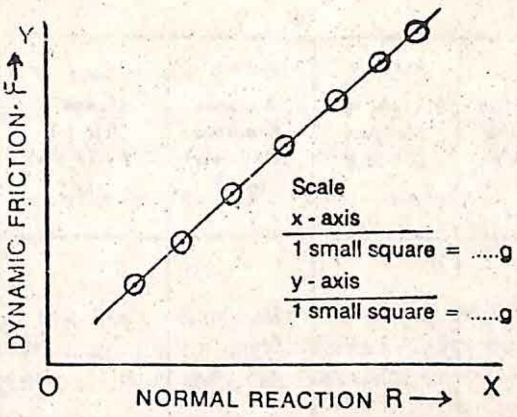


Fig. 15.1

take an observation with block alone. Similarly, the maximum normal reaction, with which you work by placing the entire set of slotted weights on the block should not be so large that its force of dynamic friction is larger than the force obtained by placing all the weights of the weight box on the pan.

2. If the slow continuous motion of the block on the glass surface is too jerky, clean the glass surface by rubbing it briskly with a clean soft dry cloth. If oil spots are seen on it, clean by soap solution.

It is essential for consistent results that the surface of any one plate shall be in a *uniform state of polish all over*. If this is not the case, the experiment must be carried over so that the portion of the lower surface moved over is always the same. This is done by marking a line on the fixed surface and starting the block always from that line.

3. It is also essential that the surfaces be in the same condition in all the observations with a given pair of surfaces. If the surfaces are pressed together before applying the force, the coefficient of friction will be changed to some extent. If moisture condenses on the surfaces, the coefficient of friction will be changed entirely.

Observations and Calculations :

Least count of the spring balance = ...g

Zero error of the spring balance = ...g

Weight of the block Observed = ...g

Corrected = w = ...g

Weight of the scale pan Observed = ...g

Corrected = P = ...g

S. No.	*Weight on the block (x) in g*	*Weight in the pan (w) in g*	*Normal Reaction R=(w+x) in g*	*Force of dynamic friction F=(P+w) in g*	*Coefficient of dynamic friction* $\mu=\frac{F}{R}$
1.					
2.					
3.					
4.					
5.					
6.					
7.					
8.					
				Mean value of μ=	

From the graph :

Force of dynamic friction (F) =.........g

Normal Reaction (R) =.........g

$\therefore$ Coefficient of dynamic friction $\mu=\frac{F}{R}$

Result :

(*i*) Since the graph between normal reaction (R) and the force of dynamic friction (F) is a straight line, therefore, $F \propto R$ for a particular pair of surfaces in contact.

(*ii*) The ratio $\frac{F}{R}$=constant and is called the coefficient of dynamic friction (μ) for a particular pair of surfaces in contact.

Precautions :

Same as in Experiment 14.

Sources of Error :

Same as in Experiment 14.

ORAL QUESTIONS

Same as in Expt. 14.

SECTION D

Experiment 16

(*a*) *To find the time-period of a simple pendulum and hence find the value of g i.e., acceleration due to gravity at a place.*

(*b*) *Draw a graph between l v/s t and l v/s* t^2 *and study which is better where l is the length of simple pendulum and t is the time period.*

(*c*) *To determine the length of the second's pendulum from the graph l v/s* t^2.

Apparatus :

Metallic bob with hook ; clamp stand ; two split pieces of cork ; fine cotton thread about 200 cm in length; Vernier callipers ; stop-watch or stop-clock ; metre-rod ; stand and a piece of chalk.

Theory :

An ideal simple pendulum consists of a heavy particle suspended by a weightless, inextensible and perfectly flexible string from a point about which it can vibrate without friction. But as these conditions cannot be achieved in practice, a metallic bob suspended by means of five cotton thread constitutes a simple pendulum.

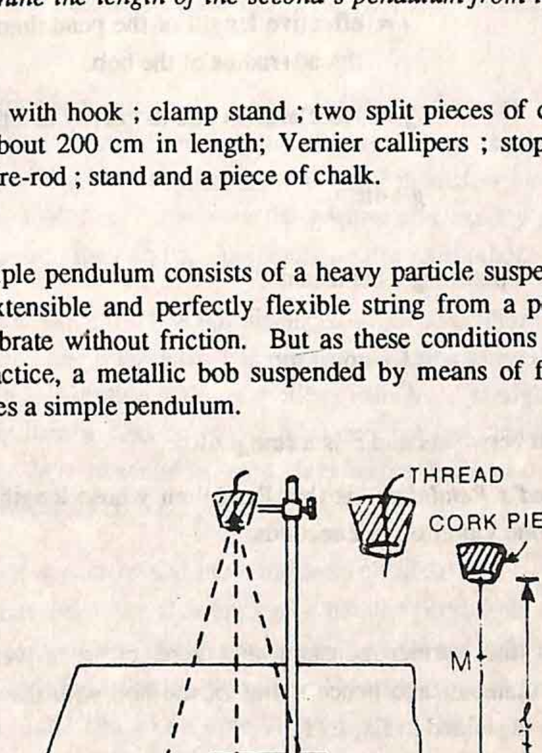


Fig. 16.1 Simple Pendulum.

As the bob is displaced from its mean position, it moves to and fro about this position with simple harmonic motion (S.H.M.). The time taken to complete one vibration (Time Period t) is given by

$$l=2\pi\sqrt{\frac{l}{g}}$$

where $l =$ effective length of the pendulum, i.e., length of thread+radius of the bob.

and $g =$ Acceleration due to gravity at a place.

$$g=4\pi^2\frac{l}{t^2}$$

At a given place, 'g' is constant

$$\frac{1}{t^2}=\text{constant}$$

or Graph between 1 and t^2 is a straight line.

A *Second's Pendulum* is that Pendulum whose length is such that it completes one vibration in 2 seconds.

Procedure :

(i) Find the vernier constant and zero error of vernier callipers. Determine diameter and hence radius of the bob with the help of vernier callipers as explained in Expt. (1)

(ii) Tie the bob to one end of the given cotton thread (150 cm). Pass other end of the thread through the two half pieces of a cork and hold the cork firmly in the clamp stand. See that cut of the cork is at right angles to the edge of the table.

(iii) Place the stand on the table as shown in the diagram above in such a way that bob is at a distance of one to two cm above the ground. Mark two lines with a piece of chalk just below the bob, one of the lines being parallel to the edge of the table and second perpendicular to it. Adjust the stand so that the bob lies exactly above the point of intersection of the two lines.

Mark two points A and B on either side of the point of intersection and at a distance of 2 cm from it as shown in the diagram. (Fig. 16.1).

(iv) Put an ink mark M with your pen on the thread at a distance of 80 cm from the centre of gravity (c.g.) of the bob. Put six more marks on the thread each at a distance of 10 cm from the previous one. This means that the new marks are at distances of 90; 100; 110; 120; 130; 140 cm from the *c.g.* of the bob.

Put the 140 cm mark just at the lower surface of the cork i.e., at the point of suspension by suitably pulling the thread through the cork piece. Now adjust the position of the clamp on the stand in such a way that the bob keeps a height of 1 to 2 cm from the ground and exactly above the point of intersection of the two lines perpendicular to each other.

(v) Take the bob along the line AB distance of 2 cm on either side of the point of intersection and release it. The pendulum begins to oscillate. See that it executes linear vibrations, i.e., it oscillates along the straight line and not along an elliptical or circular path. Also see that the pendulum does not start spinning. *Now examine the stop clock/watch and find out its least count. Also determine its zero error if any.*

If the vibrations are linear and the amplitude of vibration is small, after few vibrations, start the stop watch or clock when the pendulum just passes through 0, the mean position of the pendulum towards any side say A and count zero. When it again passes through '0' in the same direction, i.e., towards A, count 1,2,3 and so on upto 20 in this way and stop the watch when the last vibration is just completed. Note the time for 20 vibrations upto the fraction of a second. Repeat three times without stopping the vibration of the bob and take the mean time. Convert this time into seconds and divide it by the number of vibrations, i.e., 20 to get the periodic time (t) of the pendulum for this length.

(*vi*) Next, reduce the length by 10 cm by pulling the thread up after slightly loosening the clamp. Also lower the clamp to keep the bob near the floor as before and take the time period for this length. In this way repeat the experiment at least six times by reducing length.

See that at least four readings should be taken when the length of the thread is more than one metre and two readings when less than 100 cm.

Calculate the value of l/t^2 for each observation and take the mean. Find the value of 'g' by multiplying this mean 4π.

(*vii*) *Determination of Length of Second's Pendulum from Graph*. Plot $l-t^2$ graph using the value of 'l' as abscissa and the corresponding value of t^2 as ordinate with suitable scales and with the least value as origin. This will be a straight line as shown in Fig.16.2.

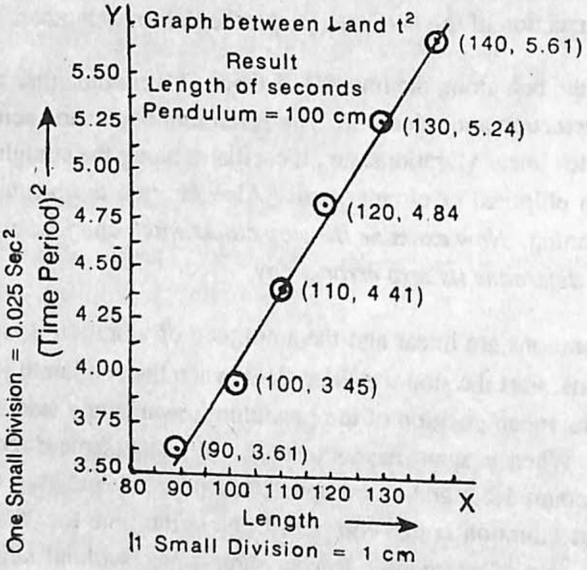


Fig. 16·2.

A simple pendulum whose line period is 2 seconds is called a second's Pendulum.

Thus if $t=2$ sec ; $t^2=4$ sec^2. Hence corresponding to $t^2=4$ along the y-axis, draw a horizontal line which intersects the graph at a point. Draw a vertical line through point of intersection and read the value of 'l', which is the required length of Second's Pendulum.

(*viii*) Plot $l-t$ graph, taking l along x-axis and t along y-axis as shown in Fig.16.3. The graph will be a curved one.

Observations :

Least count of stop watch=.........s

Zero error of stop watch=.........s

Vernier constant of the callipers (V.C.)=.........cm

Zero error of the callipers (1)=.........cm

(2)=.........cm

(3)=.........cm

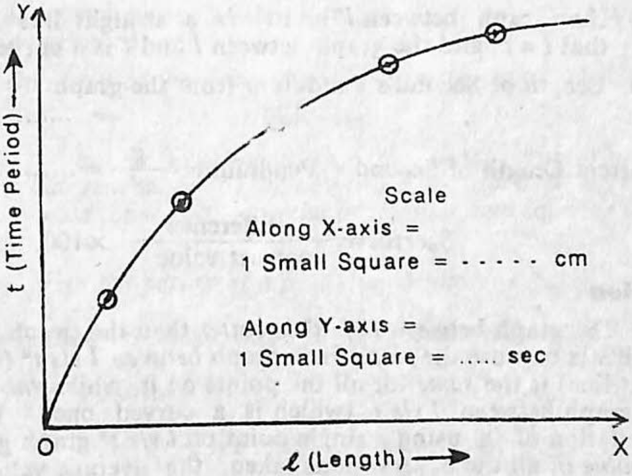


Fig.16·3. (graph l v/s t)

Mean zero correction of the callipers=.........cm

Diameter of the bob (1).........cm, (2)=.........cm

Mean=.........cm

Mean corrected diameter (d)=.........cm

Mean radius of the bob=$r=d/2$=.........cm

S. No.	*Length of the thread with hook (h) in cm*	*Effective Length $l=(h+r)$ cm*	*Corrected Time for 20 vibrations in sec*				*Time Period $t=\frac{T}{20}$ in sec*	*t^2 in sec^2*	*$\frac{l}{t^2}$ in cm/sec²*	*Mean $\frac{l}{t_2}$ in cm/sec²*
			1	*2*	*3*	*Mean=(T)*				
1										
2										
......										
8										

Calculations :

$$\therefore \qquad g=4\pi^2\frac{l}{t^2}=\ldots\ldots\ldots\ldots \text{cm/sec}^2$$

Result :

(*a*) Corrected value of 'g' = ...cm/sec^2.

Actual value of '*g*' at a place = ...cm/sec^2

$$\therefore \qquad \%\ \text{error}=\frac{\text{difference}}{\text{Actual value of '}g\text{'}}\times 100$$

(*b*) The graph between l and t^2 is a straight line showing that $l \propto t^2$ and the graph between l and t is a curved one.

(*c*) Length of Second's Pendulum from the graph

= ....cm

$$\text{Correct Length of Second's Pendulum}=\frac{g}{\pi^2}=\ldots\ldots\ldots\ldots \text{cm}$$

$$\%\text{error}=\frac{\text{difference}}{\text{correct value}}\times 100.$$

Discussion :

(*d*) The graph between l v/s t^2 is *better* than the graph between l v/s t. This is because the *slope* for a graph *between* l v/s t^2 (which is a straight line) is the *same* for all the points on it while *this is not so* for a graph between l v/s t (which is a curved one). Whereas the calculation of '*g*' using a single point on l v/s t^2 graph gives the *average value* of all the observations taken, the average value of *g* from the l v/s t graph can be found by taking *large number of* points on it as it is a curved one and the slope is different for different points.

Precautions

1. The support should be rigid and the lower faces of the cork pieces should be in the same plane.

2. The thread should vibrate at right angles to the split and should be as long as possible.

3. The amplitude should be small and the bob should move in a straight line.

4. Time should be measured carefully correct upto the smallest value.

5. The length should be changed by at least 10 to 15 cm to have a considerable change in the time period.

6. A free hand smooth graph should be drawn.

7. The experiment should be performed at a place where disturbance due to air is minimum.

Sources of Error

(*i*) Non-rigidity and vibration of suspension support.

(*ii*) Elasticity of the thread.

(*iii*) Personal error in starting and stopping the clock.

(*iv*) The inaccuracy of time measured by a stop clock or watch and presence of air currents. Any error in t is doubled in calculation of t. To measure 't' accurately the clock may be compared to a standard chronometer and the time for a large number of vibrations should be determined.

(*v*) Spinning of the bob cannot be completely avoided.

Exercises

Q. 1. *Prove by experiment that the time period of a simple pendulum is independent of* (*a*) *the amplitude provided it is not too large* (*b*) *the mass and the material of the bob, and* (*c*) *the nature of string used.*

Q. 2. *Find the period of a pendulum 5 metres long.*

Hint : Find the mean value of $\frac{l}{t^2}$ for some suitable lengths.

Let it be 'c', then

$$\frac{l}{t^2}=c \text{ or } t^2=\frac{l}{c} \text{ or } t=\sqrt{\frac{l}{c}}.$$

By substituting the value of 'c' and $l=500$ cm, 't' may be calculated.

ORAL QUESTIONS

Q. 1. *Distinguish between 'g' and 'G'. Give their units.*

Ans. 'g' is the acceleration due to gravity caused due to force of attraction due to earth. 'G' is the universal gravitational constant and it is defined as the force of attraction between two bodies of unit mass each separated by a unit distance between them.

	C.G.S. System	S.I. System
'g'	cm/sec²	m/sec²
'G'	dynes cm²/gm²	N . m²/kg²

Q. 2. *Is there any relation between 'g' and 'G'? What are the values of 'g' and 'G'?*

Ans. $g=\frac{GM}{R^2}$ where M=mass of the earth

R=Radius of the earth.

$g=980$ cm/sec² or 9·8 m/sec²

$G=6{\cdot}67\times10^{-8}$ dynes cm²/gm² or $6{\cdot}67\times10^{-11}$ N m²/kg²

Q. 24. *What is an isochronous motion ?*

Ans. It is that motion in which the time period remains the same although the amplitude goes on decreasing.

Q. 25. *Why does the amplitude decrease with time ?*

Ans. Due to resistance of air and friction at the point of support.

Q. 26. *Does the law of conservation of energy hold good during the motion of a Simple Pendulum ?*

Ans. Yes, it holds good.

Q. 27. *What will happen if the bob is made to oscillate in water ?*

Ans. The time period will increase as the effective value of 'g' decreases due to upward thrust of water.

Q. 28. *What will happen if the bob vibrates in vacuum ?*

Ans. The time period decreases as the value of 'g' increases because in vacuum there will be no effect of buoyancy of air.

Q. 29. *Why we start taking time period after it has completed some vibrations ?*

Ans. The first few vibrations are supposed to be forced vibrations. Thus the bob will not have its free time period during these few vibrations.

Q. 30. *What happens if the bob has rotatory motion along with the translatory motion ?*

Ans. There will be twists in the thread due to rotatory motion which will affect the time period.

Q. 31. *Why do the clocks go fast in winter and slow in summer ?*

Ans. Because the length of the pendulum decreases in winter, so the time period decreases and thus the clocks go fast in winter. In summer their length increases and thus the time period increases resulting in the slow running of the clock.

Q. 32. *Why are the pendulums of good clocks made of Invar ?*

Ans. Because Invar (an alloy containing 64% of iron and 36% of nickel) has a very small coefficient of expansion.

Q. 33. *What is personal error ?*

Ans. The stop clock may not be started or stopped at the correct time, thus introducing on error in noting down the time period.

Q. 34. *What is an inaccessible pendulum ?*

Ans. It is that pendulum whose point of suspension is so high that its length cannot be measured directly.

Q. 35. *What type of graphs we get between (a) l and t^2, (b) l and t?*

Ans. (*a*) Straight line. (*b*) Parabolic curve.

Q. 36. *What is the principle underlying measurement of time ?*

Ans. In principle any process which repeats itself after regular intervals of time can be used for measuring time.

Q. 37. *Upon what factor does the pulse rate of a human being depend ?*

Ans. Pulse rate depends upon the mental and physical conditions and it varies from one individual to another.

Q. 38. *How could pulse rate be used for measuring time ? Is it reliable method ?*

Ans. By calibrating one's own pulse beat. Since pulse rate varies from one individual to other and it depends upon the mental and physical conditions so as such a lot of subjectivity creeps into the measurement.

Hence it is not a reliable method.

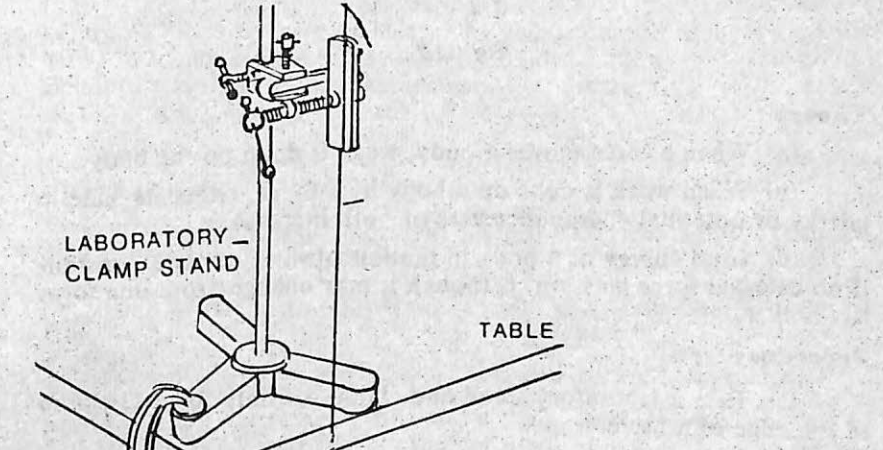
Experiment 17 :

To study the conservation of energy of a simple pendulum using ticker timer and a tape.

Apparatus :

A brick, laboratory clamp stand, four G-clamps, ticker tape time, strings, ticker tape, string of length 2 m, spring balance.

Diagrams :



To ticker timer placed at the same level as the lower surface of the brick.

Fig. 17.1.

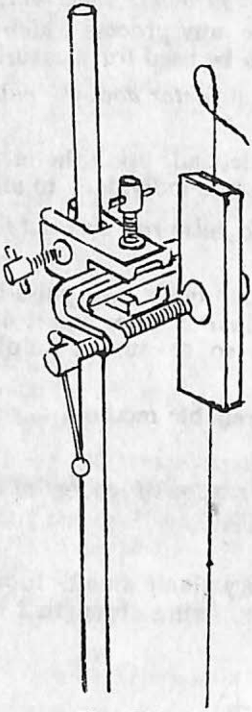


Fig. 17.2.

Theory :

(*a*) When a force moves a body, work is done on the body.

(*b*) When work is done on a body by a force, either its kinetic energy or potential energy increases or both increase.

(*c*) Total energy of a body in motion always remains constant if no external force acts on it though it may change from one form to another.

Procedure :

1. Take a laboratory stand and clamp it firmly by a G-clamp to the edge of a table.

2. Make a heavy pendulum bob by tying a string to a brick. Hang it to the clamp stand as shown in Fig. 17.1.

The more massive the support the more successful is the experiment. A light support allows energy to leak away. So does a loose support. Therefore, the pendulum cord should be firmly clamped at the top between two bits of metal or wood with their lower edges flush as shown in Fig. 17 2.

3. To avoid the toppling of the stand, reinforce the vertical rod of the stand by tying its upper end with wires of soft iron to three G-clamps fixed on the edges of the table.

4. Adjust the length of the string such that the total length of the pendulum (*i.e.*, the length from the point of suspension to the centre of gravity of the heavy bob) is about 1·5 m.

5. Tie another string or thread through the middle of the brick such that the line of the thread passes through the c.g. of the brick.

6. Put the timer at about the same level as the lower surface of the brick. Attach the tape of the timer to the brick. Pull the thread passing through the c.g. aside such that the displacement of the brick is not more than 10° from the vertical. Hold the string there. The tape of the timer should also be taut.

7. Start the timer and let the thread go. As the brick moves to the other end, it pulls the tape through the timer and makes a record of the position of the brick at successive time intervals. When the brick reaches the far end switch off the timer.

On the record of the tape the extreme positions of the pendulum bob (brick) are represented by the two extreme dots on the tape. The central position of the pendulum is represented by the centre of the two extreme dots. Moreover, another check of the point on the tape corresponding to the central position of the pendulum is that at equal distances on either side of the central point, spacing among adjacent dots is equal.

8. Measure the displacements of the pendulum from the central point to 12 or so selected dots.

9. Also find the time when each of the selected dot was made, by counting dots from the central point.

10. Draw a graph of the displacement of the brick against time.

11. Find the mass of the brick with a spring balance.

12. Use the graph to find the velocity of the pendulum at 4 or 5 points on the left of the central position and 4 or 5 points on the right. Now calculate the kinetic energy (K.E.) at these points.

13. Plot a graph of the K.E. of the pendulum versus its position. Find out the point at which it is minimum.

14. The potential energy (P.E.) of the pendulum, at these points on which K.E. was calculated, can also be computed by knowing the height through which the brick is lifted above the

central position. The P.E. at the central position is taken to be zero. The height can be found by a simple relation.

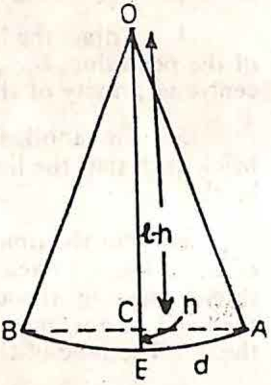


Fig. 17.3.

If l is the length of the pendulum and d its horizontal displacement from the central position, then, the height h above the rest position (Fig. 17.3) is found as follows : $OC \times CE = AC \times CB$

$$(l-h) \times h = \sqrt{d^2-h^2} \times \sqrt{d^2-h^2}$$

$$lh-h^2 = d^2-h^2 \qquad \therefore \quad lh = d^2$$

or $$h = \frac{d^2}{l}$$

(This holds as long as d is small compared to l).

15. Draw a graph of P.E. versus position on the same graph on which K.E. versus position was drawn. Study the change in P.E. and K.E. and see how do they compare.

16. Find the sum of the K.E. and P.E. of the pendulum at the selected points.

(If desired, the experiment can be made shorter by omitting the graphs for kinetic and potential energies. Instead total energy at the selected points may be calculated.)

Observations and Calculations :

Zero error of the spring balance=........g

Mass of the brick

Observed=........g Corrected=M=.. .....g

Side from Centre	S. No.	Displacement (d) in m	Time (t) in s	Velocity $v=\frac{d}{t}$ in m/s	V^2 in $(m/s)^2$	K.E. $=\frac{1}{2}mV^2$ in J	Height (h) in m	P.E. $=mgh$ in J	K.E. +P.E. in J
Left	1.								
	2.								
	3.								
	4.								
Right	1.								
	2.								
	3.								
	4.								

Result :

The sum total of mechanical energy (K.E.+P.E.) possessed by the simple pendulum at any point always remains constant within experimental error. Energy is neither being created or destroyed, it is being transformed from K.E. to P.E. and vice-vessa with exact equivalence. Any increase in K.E. is exactly equal to the decrease in P.E. and vice-versa.

Precautions :

1. The displacement of the brick should not exceed 5° to 10° in order that (*i*) formula for calculation of *h* is valid and (*ii*) distances of dots on tape from central point truly represent corresponding displacements of the brick from its central position.

2. Length of the pendulum should be at least 1·5 cm.

3. The support should be rigid and massive.

4. Spinning of the bob (brick) must be avoided.

5. There must not be energy losses due to light and loose support.

ORAL QUESTIONS

(Same as in Expt. 6).

Experiment 18 :

To study the conservation of energy of a body falling freely using ticker time and tape.

Apparatus :

Ticker-timer, tape ; pulley or glass rod for paper tape to pass over it, an object of mass about 100 g (which can be easily suspended by the tape with a sellotape).

Theory :

Same as in Expt. 17.

Procedure :

1. Clamp the ticker timer near the edge of the stool placed on the table so that the paper tape, after passing over the glass rod (or a glass tube or pulley) hangs vertically. The glass rod is used to reduce friction and save the tape from tearing.

2. Hang the object with the tape using a sellotape (Fig. 18.1). Hold the spool of the tape in a horizontal axle clamped in a stand so that it feeds the tape into the timer without applying any significant drag while the body is falling freely.

3. Hold the body at rest by pressing the tape on to the open portion of the ticker-timer by the thumb. Start the ticker-timer and release the tape. (*It is advisible to cut off the appropriate length of tape from the reel before dropping the weight.*)

Diagram

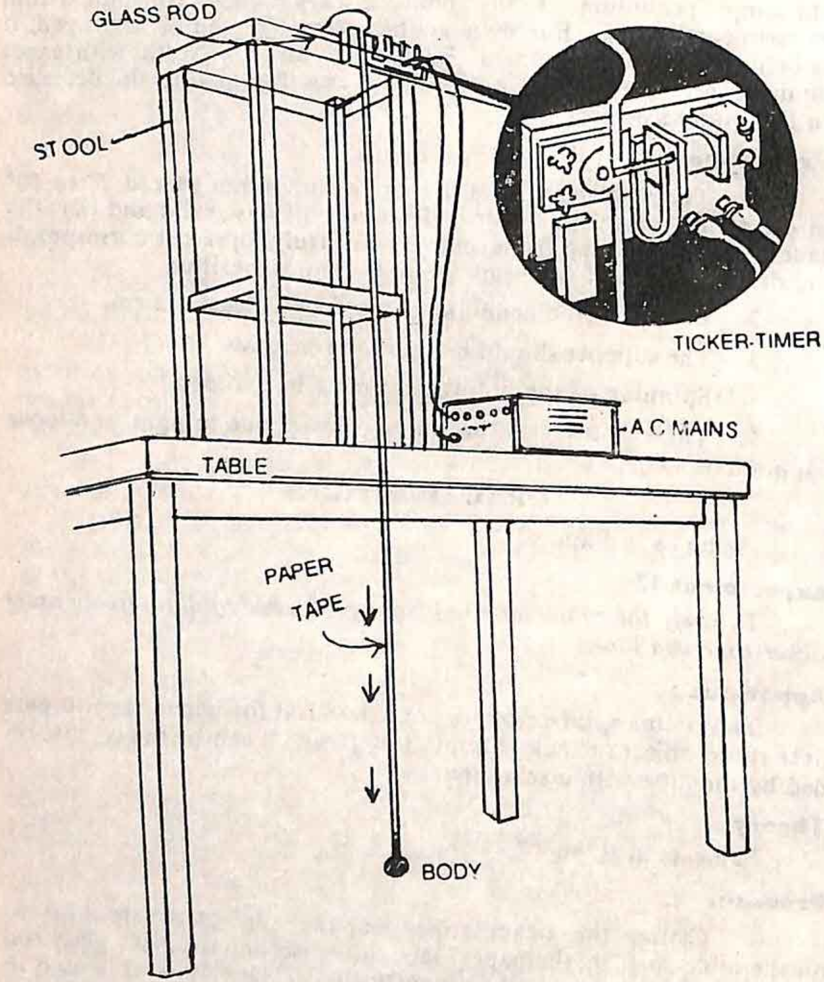


Fig. 18.1.

4. Find the velocity of the object at different heights above the ground (*i e.* at a few selected dots on the tape corresponding to which height of the object above ground can be measured on the tape). Consider an equal number of dots, n, before a selected dot and after the selected dot. The distance covered by the object between these $(2n+1)$ dots, S, is equal to the length of the tape between first and last of these dots. The time taken for moving the distance is equal to $2n$ times the period of the timer. Thus the velocity, V,

of the object at the selected dot is $\frac{S}{2nT}$ where T is the time-period of the timer.

5. Calculate the K.E. of the body at each height using the formula K.E.$=\frac{1}{2}mv^2$ where m is the mass of the object.

Also calculate the P.E. at all these heights for which K.E. has been calculated by using the formula, P.E.$=mgh$ where h is the height of the object above the ground.

6. Compare the K.E. with the P.E. at each dot. You may find that their sum is constant within experimental error.

Notes :

(*i*) Leave a few dots in the beginning of the motion, since they will be too close to each other and their analysis may be difficult.

(*ii*) After the body reaches the ground, mark a point on the tape below the vibrator of the timer. You need it for finding the height of the object above ground for various selected dots. But your last dot selected for finding the velocity of the body must be n dots before it.

Observations and Calculations :

Time period of the timer=.........seconds

Dot No.	*Height (h) above ground in m*	*Velocity $V=\frac{S}{2nT}$ in ms^{-1}*	*K.E. $E=\frac{1}{2}mv^2$ in J*	*P.E. $E_p=mgh$ in J*	*Sum=K.E. (E_k) +P.E. (E_p) in J*

Result :

The sum total of K.E. and P.E. of the body falling freely under gravity is always found to be constant within experimental error at any point during its fall. Any increase in K.E. is exactly equal to the decrease in P.E. and vice-versa.

Precautions :

1. Clamp the ticker-time near the edge of the stool placed on the table properly so that the paper tape after passing over the glass rod hangs *vertically*.

2. There should not be any *drag* on the paper tape as the body falls freely under gravity.

Note. Results will not be very consistent because the acceleration is large and there will be trouble due to friction.

ORAL QUESTIONS

(Same as in Expt.16).

Experiment 19 :

To study the conservation of energy of a ball rolling down an inclined plane (using a double inclined track).

Apparatus :

Double inclined track, steel ball of about 2·5 cm diameter, two wooden blocks each about 2 cm high ; two weights of 1 kg each ; spirit level.

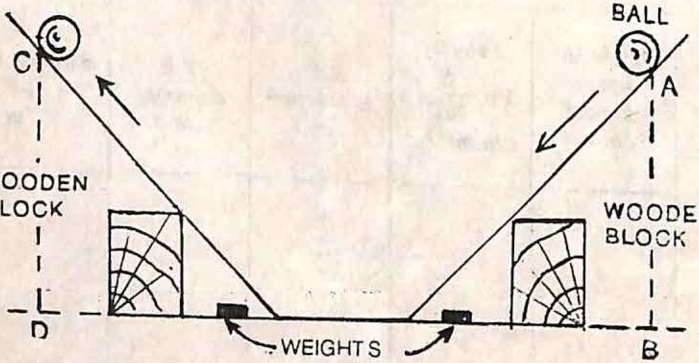


Fig. 19.1.

Theory :

Same as in Experiment 17.

Procedure :

1. First adjust the table horizontal.

2. Keep the double inclined track on the table and press its central wings with the help of 1 kg weights (Fig. 19.1). Insert the wooden blocks under the ends of each track such that both are inclined. The angle of inclination need not necessarily be equal.

[You can also use flexible curtain rail as shown in Fig. 14.2, as an alternative set up for this experiment].

3. Release the ball from any point, A, say 50 cm mark on the right track. Notice the mark *c* upto which the ball reaches on the left track. Find the vertical height AB of the 50 cm mark from the table. Also find the vertical height CD, the height to which the ball reaches on the left track.

4. Repeat the experiment for various positions of starting point of the ball.

Observations and Calculations :

S *No*	*Starting Point A on the right track*	*Reach of the ball on the left track C*	*Vertical height of A above centre=AB (in cm)*	*Vertical height of C above centre=CD (in cm)*	*Difference AB−CD*
1.					
2.					
3.					
4.					
5.					

Result :

Since $AB \approx CD$ within experimental errors so the energy of the ball rolling down an inclined plane is always conserved. Energy is being transformed from P.E. to K.E. and vice-versa with exact equivalence.

Precautions :

Same as in Experiment 13.

ORAL QUESTIONS

(Same as in Expt.16).

Experiment 20 :

To study the variation in the range of a jet of water by varying the angle of projection.

Apparatus :

Rubber or PVC pipe, a nozzle to obtain jet of water, arrangement for water under pressure from the tap or tank ; a measuring tape and a protractor.

Note. A good venue for this experiment is the garden at your school or home where the source of tap water and hose-pipe are available and you can throw a jet of water to various distances.

Theory :

A projectile may be defined as a body which after having been given an initial velocity is allowed to move under the action of gravity alone. Imagine a body projected upward with an initial speed V_0 at an angle θ to the horizontal (Fig. 20.1). Its path is a *parabola*. At the instant t, the x-coordinate is the horizontal range R. The maximum horizontal range is realized when $\theta = 45°$.

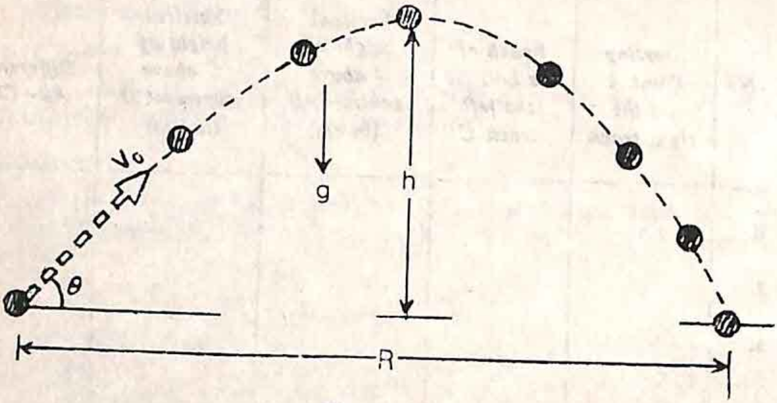


Fig. 20.1.

Procedure :

1. Fix one end of pipe to a tap. At the other end of the pipe, fix the nozzle and obtain jet of water.

2. Put the pipe flat on the ground. Now raise the front portion of the pipe slightly so that the water projects at an angle say 15° to the ground. Notice the range of the jet of water.

3. Now increase the angle of projection of water by 15°. Note the range of water jet.

4. Repeat the experiment for four more inclinations—increasing each time by 15°.

5. Plot the variation of range with the angle of inclination of pipe.

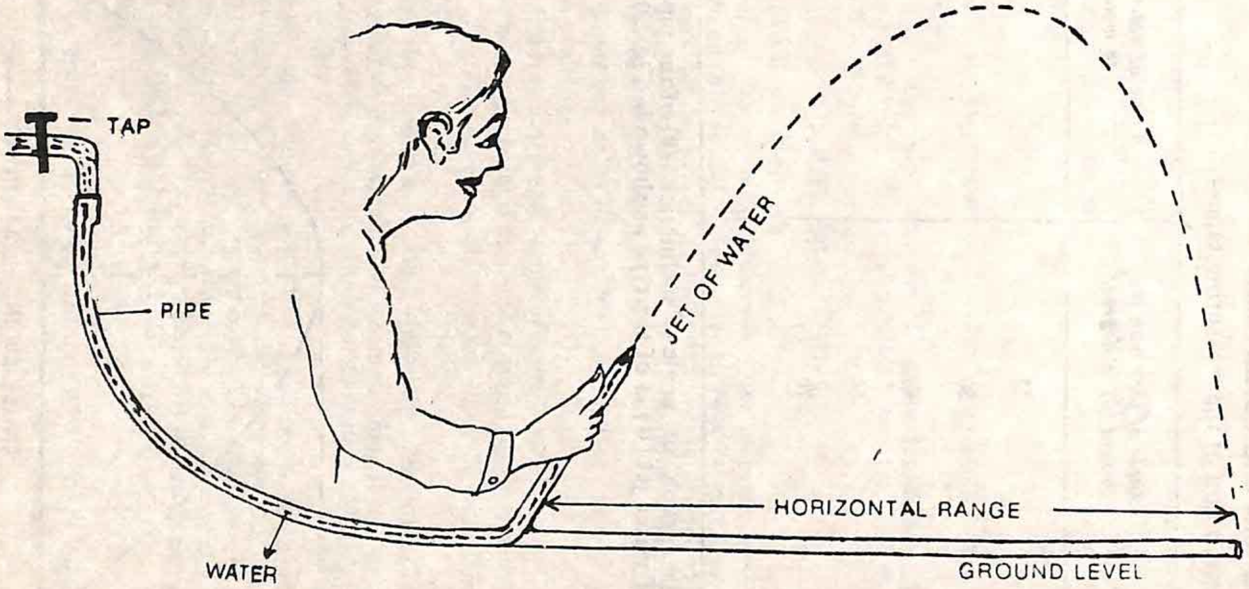


Fig. 20.2.

Observations and Calculations :

Least count of the measuring tape=.........

S. No.	*Angle of pipe with the ground (θ) in degrees*	*Range of water jet (R) in metre*
1.	15	
2.	30	
3.	45	
4.	60	
5.	70	
6	80	

The graph of angle of inclination (θ) of the pipe against the range of water jet (R) is of the type as shown in Fig. 20.3

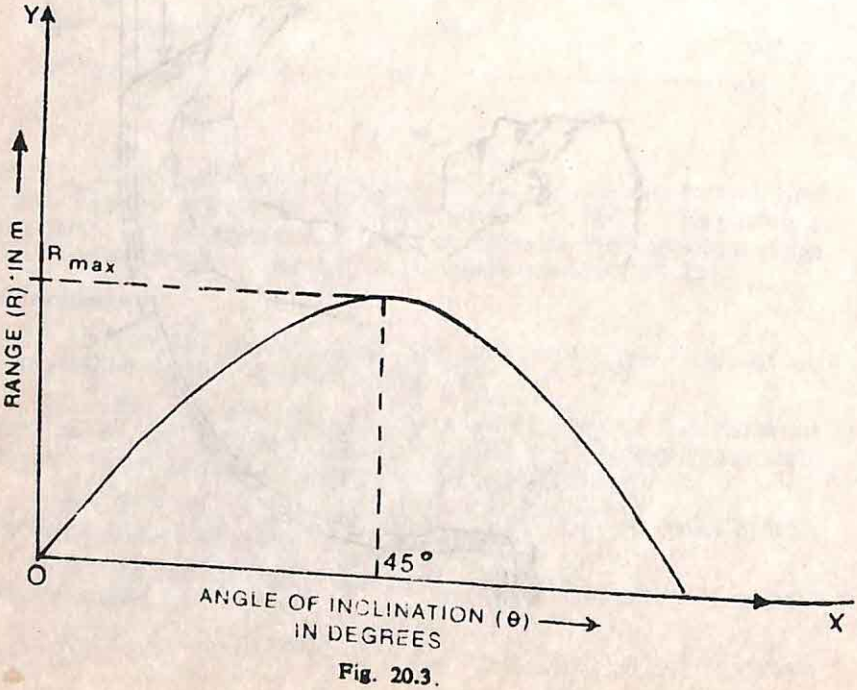


Fig. 20.3.

Result :

The horizontal range of water jet increases with the increase in angle of inclination of the pipe with the ground, becomes maximum when the angle of inclination of the pipe with the ground is 45° and then starts decreasing as the angle of inclination of the pipe increases beyond 45°. The horizontal range is maximum when $\theta = 45°$

The maximum horizontal

Range ($R_{max.}$)= ....m

Precautions :

1. Water under pressure is necessary to obtain a jet of water through a nozzle.

2. First put the pipe flat on the ground and then raise the front portion of the pipe to get the desired angle of inclination of the pipe with the ground.

ORAL QUESTIONS

Q. 1. *What is a projectile ?*

Ans. A projectile may be defined as a body which after having been given an initial velocity is allowed to move under the action of gravity alone *i.e.*, it is no longer being propelled by any fuel *e.g.*, a javelin throw by an athlete, a bomb released from an aeroplane, a bullet shot from a rifle, etc.

Q 2. *Name the path followed by a projectile.*

Ans. A parabola.

Q 3. *What is horizontal range of a projectile ?*

Ans. It is the horizontal distance covered by the projectile.

Q 4. *If a man wants to hit a target, in what direction should he point his rifle ?*

Ans. He should aim at a point *higher* than the target.

Q 5. *A bullet is dropped from the same height and at exactly the same time, another bullet is fired horizontally. Which one will hit the ground earlier ?*

Ans. Both the bullets will hit the ground *at the same time*. The horizontal velocity of the second bullet does not interfere with the vertical velocity produced by gravity. The two are quite *independent* of each other. Therefore, although the path of the second bullet is a *parabola*, yet the time taken by it to reach the ground will be the *same* as that of the first bullet.

Experiment 21 :

To plot a graph between the distance of the knife edges from the centre of gravity and the time period of a bar pendulum. From the graph find

(*a*) *the acceleration due to gravity at*........*

---

*Name the place where the experiment is being conducted.

(b) the radius of gyration of the bar about an axis passing through the centre of gravity and perpendicular to its plane.

Apparatus Required :

The bar pendulum, knife-edge for suspending the pendulum, metre scale, spirit level and a stop-watch.

Description of the Apparatus :

A simple form of compound pendulum designed by D. Owen in 1939 is shown in the accompanying figure 21.1. It consists of a metallic bar nearly a metre long, in which a series of circular holes of nearly 5 mm, diameter are bored at equal distances (nearly 2 cm), along its length. With the help of these holes the bar can be suspended from a knife-edge and made to oscillate. The knife-edge is fixed in a platform supported on three screws ; the hinder one of which is adjustable, thereby the platform can be made horizontal.

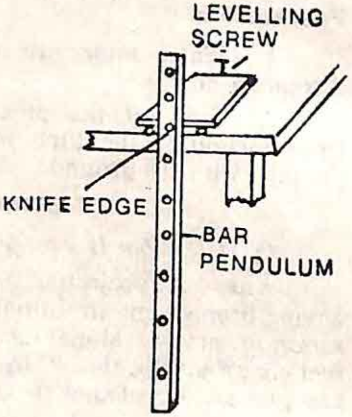


Fig. 21.1.
Bar pendulum.

Formula Employed :

The value of g can be calculated from any of the following formula.

$$g=\frac{4\pi^2}{T_0^2}\cdot(2k) \qquad (1)$$

The minimum time-period T_0, and $2k$ (where k is the radius of gyration of the bar about an axis passing through its centre of gravity and parallel to the axis of rotation) can be read from the graph.

Also,
$$g=4\pi^2\cdot\frac{h_1+h_2}{T^2} \qquad ...(2)$$

where T=time-period

h_1+h_2=length of the equivalent simple pendulum.

Both these quantities can be read off from the graph.

The radius of gyration can also be calculated from the formula :

$$k=\sqrt{h_1 \cdot h_2} \qquad .(3)$$

PRINCIPLE AND THEORY OF THE EXPERIMENT

If the compound pendulum be allowed to oscillate about a horizontal knife-edge passing successively through each hole, and a graph be plotted taking the periods of oscillation as ordinates and corresponding distances of the axis of suspension from the C. G. of the bar as abscissae, a graph of the type shown below (Fig. 21.2) will be obtained :

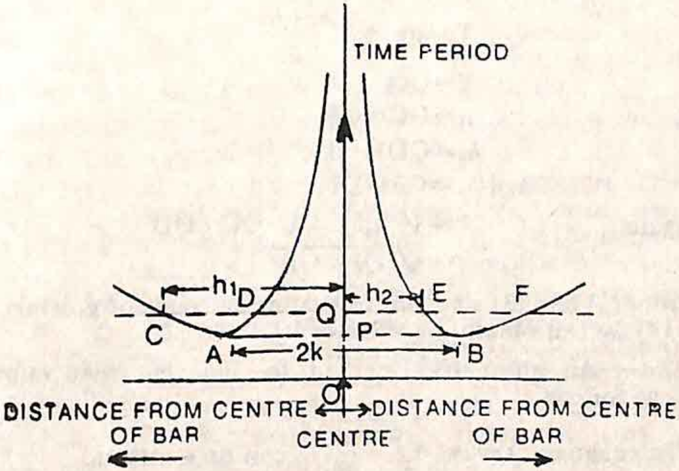


Fig. 21.2.

T—h graph for a compound pendulum.

Now we know that the time-period T of a compound pendulum is given by :

$$T=2\pi \cdot \sqrt{\frac{k^2+h^2}{hg}}$$

where h is the distance of its centre of gravity from the point of suspension, and k is the radius of gyration about a parallel axis passing through the centre of gravity.

When the axis of suspension passes through the centre of gravity, the periodic time becomes infinitely great. If the axis is at an infinite distance the periodic time is again infinite. Consequently there must be some intermediate position for which the periodic time is a minimum. Now T will be a minimum when $\frac{k^2+h^2}{h}$ is minimum.

But $$\frac{k^2+h^2}{h}=\frac{(k-h)^2+2kh}{h}=\frac{(k-h)^2}{h}+2k$$

This is clearly a minimum when $k=h$.

Thus the minmum time-period

$$T_0=2\pi \sqrt{\frac{k^2+k^2}{kg}}=2\pi \sqrt{\frac{2k}{g}} \quad ...(1)$$

From the graph, $T_0=OP$

and $2k=AB$ ...(2)

Now any line drawn parallel to the distance axis will cut, in general, the curve in four points such as C, D, E and F which are situated symmetrically about the time-axis. If T be the periodic time corresponding to these points, then

$$T=2\pi\sqrt{\frac{h_1+h_2}{g}} \qquad \text{...(3)}$$

where
$$T=OQ$$
$$h_1=QC=QF$$
$$h_2=QD=QE$$
or
$$h_1+h_2=CE=DF$$

Again
$$k=\sqrt{h_1h_2}=\sqrt{QC\,.\,QD}$$
$$=\sqrt{QF\,.\,QE} \qquad \text{...(4)}$$

From (1) and (3) we can calculate the value of g, while from (2) and (4) we can obtain the value of k.

[**Note**—An alternative method to find the mean value of g and k is as follows :

The equation $T=2\pi\sqrt{\frac{k^2+h^2}{hg}}$ can be written as

$$T^2h=\frac{4\pi^2}{g}\,.\,h^2+\frac{4\pi^2}{g}\,k^2$$

By plotting T^2h against h^2, a curve (straight line) as shown in

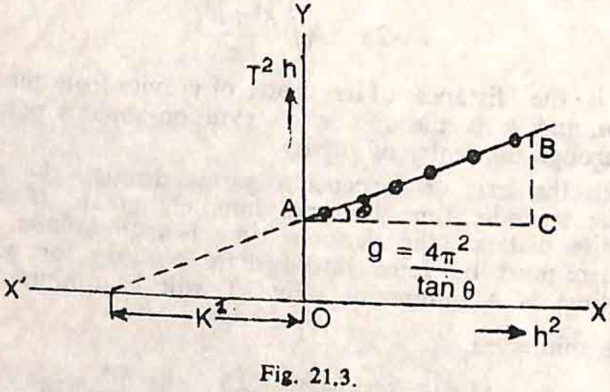


Fig. 21.3.

Fig. 21.3 is obtained. The slope of this curve is given by

$$\tan\theta=\frac{4\pi^2}{g}$$

Hence
$$g=\frac{4\pi^2}{\tan\theta}$$

The intercept on the x-axis gives directly the value of k^2 whence the radius of gyration (k) can be evaluated.]

Procedure :

(*i*) First of all make the knife-edge horizontal with the help of the levelling screw provided with the platform and test the horizontality with a spirit level. Suspend the pendulum about the knife-edge from the hole nearest one end. Displace the bar *slightly* and release it so that it begins to oscillate in a vertical plane. Note the time with a stop-watch for a known number of oscillations* and from this calculate the periodic time.

(*ii*) In this way determine the periodic time of the bar when it is successively suspended† from the holes.

(*iii*) Now with the help of a metre scale measure the distance of the positions of the knife-edge from the C. G.* of the bar. Then plot a graph between the periodic times (T) and the corresponding distances (h) of the points of suspension from the C. G. of the bar.

(*iv*) Join A and B (See Fig. 21.2). The line AB cuts the T-axis at P. The abscissa of P gives the position of the centre of gravity of the bar. Draw any line CDEF perpendicular to the T-axis and cutting the curve in four points C, D, E and F. Measure QC and QF (the mean of which gives h_1), QD and QE (the mean of which gives h_2), and also OQ (which gives the corresponding periodic time T. Then from the formula (3) given above calculate the value of g.

Again, with the help of formulae (2) and (4) given above calculate the value of k, the radius of gyration of the bar.

Observations :

Readings for the Measurement of h and T

Least count of the stop-watch= .... sec

No. of the hole	*Distance of hole form thc C. G. (h)*	*No. of Oscillations*	*Time taken*	*Periodic Time (T)*
1		25 25 25 25	........sec ....sec ....sec ....sec	........sec
2 ⋮				

---

*To facilitate the counting of oscillations correctly, place a pointer (or make a mark on the wall behind the bar) coincident with the mean position of the pendulum.

†As the pendulum is made to oscillate from the holes which are near to the centre, the time period increases, hence very few oscillations can be timed in these positions. When the bar is suspended from the central hole it may not even be set to vibration, since its weight does not have any moment about the fulcrum which now passes through the centre of gravity of the bar.

*Note the position of the C.G. by balancing the bar on a sharp wedge.

Calculations :

From the graph

(*i*) $\left\{\begin{array}{l} QC = \ldots\ldots\ldots \text{ cm} \\ QF = \ldots\ldots\ldots \text{ cm} \end{array}\right.$ $\therefore$ Mean $h_1 = \ldots$ cm

$\left\{\begin{array}{l} QD = \ldots\ldots\ldots \text{ cm} \\ QE = \ldots\ldots\ldots \text{ cm} \end{array}\right.$ $\therefore$ Mean $h_2 = \ldots$ cm

$\therefore h_1 + h_2 = \ldots\ldots\ldots$ cm

Also $T = OQ \equiv \ldots\ldots\ldots$ sec

$$\therefore \quad g = 4\pi^2 . \frac{h_1 + h_2}{T^2}$$

$= \ldots\ldots\ldots \text{cm/sec}^2$

(*ii*) $2k = AB = \ldots\ldots\ldots$cm

$T_0 = OP = \ldots\ldots$sec

$$\therefore \quad g = 4\pi^2 . \frac{2k}{T^2}$$

$= \ldots\ldots\ldots \text{cm/sec}^2$

(*iii*) $k = \sqrt{h_1 h_2}$

$= \ldots\ldots\ldots$ cm

Also $k = \frac{1}{2}$ AB$= \ldots\ldots\ldots$ cm

Result :

The graph depicting the relation between the time-period (T) of a compound pendulum and the distance (h) of the point of suspension from the centre of gravity is attached herewith.

(*i*) The radius of gyration of the bar about an axis through its centre of gravity and perpendicular to its plane$= \ldots\ldots\ldots$ cm

(*ii*) The value of g at...... $= \ldots\ldots\ldots \text{cm/sec}^2$

Precautions and Sources of Error :

1. Before starting the experiment make the knife-edge horizontal. This adjustment will keep the pendulum oscillating in a vertical plane, and secondly the bar shall not tend to slip off the knife-edge during oscillations.

2. In the theoretical deduction of the formula it has been assumed that $\sin\theta = \theta$ where θ is the angular deflection of the bar. Hence to satisfy this condition the bar should not be displaced more than 5° from its mean position.

3. To get very exact points on the curve in the vicinity of the minimum period, the time should be observed very carefully. Try to note the time for 100 oscillations except for points very close to the centre of gravity of the bar, where, due to large time periods, few oscillations can be timed.

4. Before taking observations see that the pendulum is oscillating in the vertical plane only, and that all other irregular motions, if any, have subsided.

5. The curves on the graph should be smoothly drawn.

6. The manner of observing the oscillations is far from satisfactory ; secondly the time period has not been corrected for (*i*) finite arc of swing, (*ii*) air effects, (*iii*) curvature of knife-edge, and (*iv*) yielding of support, hence the result is not free from errors due to these causes.

ORAL QUESTIONS

(*Also read Oral Questions in Expt.*16)

Q. 1. *What is a compound pendulum ?*

Ans. A compound pendulum is just a rigid body capable of oscillating freely about a horizontal axis passing through it.

Q. 2. *If any rigid body can serve the purpose of a compound pendulum, then why this particular shape of your instrument (bar or metre scale) ?*

Ans. There is one serious disadvantage of a compound pendulum. Due to its vibration some air is dragged with it, thus increasing its effective mass and hence its moment of inertia. But it has been shown by Bessel that if it be of a form symmetrical about the centre of geometrical shape,* this error is automatically eliminated. This is one of the reasons of using a compound pendulum in the shape of a bar or a metre-scale.

Secondly this bar or metre-scale has been provided with circular holes situated at equal distances on either side of its centre of gravity. By slipping any hole on to a horizontal knife edge, the bar can be made to oscillate about it in the vertical plane. Thus we can study the variation of the time period with the distance of the axis of rotation from the centre of gravity of the bar.

Q. 3. *Does the time period vary ? How does it vary ?*

Ans. Yes. It does vary. The time-period of the pendulum is given by the formula

$$T=2\pi\sqrt{\frac{\frac{k^2}{h}+h}{g}}$$

where k is the radius of gyration of the bar and h is the distance of the point of suspension from the centre of gravity of the bar.

Thus if a graph is plotted between T and h, its nature is as shown in Fig. 21.2. It is clear from the graph that there are two values of h on either side of the C.G. which give the same value of T.

Q. 4. *Explain what will happen if the pendulum is made to oscillate about its C.G. ?*

Ans In the formula for T if we put $h=0$ (since the axis of rotation passes through the C.G.), the time-period assumes an infinite value. This can also be visualized from the fact that under this circumstance the weight of the pendulum has no component about the fulcrum, hence it cannot be set into vibration, or, which is the same thing the time-period becomes infinite.

---

*Students should bear in mind that the centre of geometrical shape of a body is not the same thing as its centre of gravity.

Q. 5. *So, according to you, there are two points on either side of the C G. about which the periods are equal. Name and define these points.*

Ans. One point is known as the *centre of suspension*, which can be defined as that point at which the axis of rotation (or the fulcrum) intersects the plane of oscillation of the pendulum.

The second point, known as *the centre of oscillation*, is a point situated on the other side of the C.G. and lying on the line joining the point of oscillation to the C.G. and distant $\frac{k^2}{l}$ from it (the C.G.).

Q. 6. *Now tell me one thing. When g can be determined with a simple pendulum, what is the advantage of using a compound pendulum ?*

Ans. There are several reasons for this :

(*i*) Whereas a simple pendulum is just an ideal conception, not realisable in actual practice in the case of a compound pendulum the length of an equivalent simple pendulum and hence the value of *g* can be easily and accurately determined.

(*ii*) The compound pendulum vibrates as a whole, there being no lag between the bob and the string.

(*iii*) In the case of a compound pendulum, the length to be measured is clearly defined and hence can be easily and accurately measured, whereas in the case of a simple pendulum, the point of suspension and the C.G. of the bob are both, more or less indefinite points, and hence its true length can hardly be expected to be accurately determined.

(*iv*) Since the compound pendulum has a large mass, it continues to vibrate for a long time, thus its periodic time can be determined with sufficient accuracy. In a simple pendulum the oscillations cease much sooner, since the mass of the bob is comparatively much less. Hence the accuracy attained in the determination of the time-period is not to the same degree as in the case of the compound pendulum.

(*v*) In the case of a simple pendulum, the string slightly slackens at the extremity of its vibration, but the compound pendulum being a rigid body, no such thing happens on this case.

Q. 7. *So it means that this is an ideal pendulum and there are no sources of error in your experiment. Is it not ?*

Ans. No, this is not the meaning of the statement. There are several sources of error in this experiment.

Q. 8. *What are they?*

Ans. (*i*) **Finite Amplitude of Swing.** The expression for the time-period has been obtained on the assumption that the angular displacement of the pendulum is vanishingly small. This is far from truth in actual practice.

(*ii*) **Air Effects.** The presence of air in the medium in which the pendulum swings affects its vibrations in three ways :

Firstly, due to the buoyancy of the air there is a slight decrease in the weight of the pendulum, thereby affecting the time-period.

Secondly, some air is dragged alongwith the pendulum. This increases the effective mass of the pendulum, thereby increasing its moment of inertia.

Thirdly, due to the viscosity of the medium, the motion of the pendulum is resisted. Consequently its time-period is affected.

(*iii*) **Non-rigidity of the support.** Due to a force being applied by the vibrating pendulum, the support may yield. This will obviously affect the time-period of the pendulum.

(*iv*) **Curvature of the knife edge.** If the knife edge is not quite sharp but rounded, an error is introduced in the time-period.

(*v*) **Change of temperature during experiment.** This results in a corresponding change in the length of the pendulum, and hence in its time-period.

Q. 9. *It means that due to the presence of these sources of error, the value of g cannot be accurately determined. Is it right?*

Ans. No, this is not exactly so. The corrections due to these causes have been theoretically studied and mathematical formulae have been deduced for the same. By applying these corrections accurate value of *g* can be obtained.

Q. 10. *All right. How will you find the value of g from these observations?*

Ans. I will draw a graph between T and *h* (Fig. 21.2) and shall get two curves of this type. Then I shall draw a horizontal line CDQEF. From this I shall read h_1 (=QC or QF) and h_2 (=QD or QE). Noting the corresponding time-period T, I shall get the value of *g* with the help of the formula.

Q. 11. *Is there any other way of calculating g with this graph?*

Ans. Yes. If we note the minimum time period T_0, then $g=\frac{4\pi^2}{T_0^2}\cdot 2k$ where $2k$ is the distance between the two minima points lying on the two branches of the curve.

Q. 12. *Can you get a straight line graph with these readings ?*

Ans. Yes. The formula for the compound pendulum is

$$T=2\pi\sqrt{\frac{\frac{k^2}{h}+h}{g}}$$

This can be put in form

$$T^2h=\frac{4\pi^2}{g}\cdot h^2+\frac{4\pi^2}{g}k^2$$

or

$$y=\frac{4\pi^2}{g}\cdot x+\frac{4\pi^2}{g}k^2$$

which is of the form $y=mx+c$. Thus by drawing a graph between T^2h and h^2, we get a straight line (Fig. 21.3).

Q. 13. *Can you get the value of g and k from this graph ?*

Ans. Yes. The slope θ of the straight line is given by $\tan\theta=\frac{4\pi^2}{g}$. From this g can be calculated. Again, the intercept of this line on the x-axis gives k^2 and hence k.

Q. 14. *Explain in a simple manner what do you understand from the expression "Moment of Inertia of a body" ?*

Ans. Every body maintains or tends to maintain its state of rest or of uniform motion in a straight line. If we wish to change this state, the body offers a kind of passive resistance to the agent moving it. This passive resistance or inertness of bodies is called *Inertia* and is proportional to the mass of the body. This is the case of a translational motion. In the case of a rotatory motion, however, the resistance offered by a body does not depend simply on its mass. The sluggishness of a rotating body is called rotational inertia or *moment of inertia*. Unlike the simple inertia of a body undergoing a motion of translation, the rotational inertia depends upon the total mass of the body and the distances of various portions of the body from the axis about which the body rotates.

Thus, in rotatory motion the quantity known as moment of inertia plays the same role as mass does in a translatory motion.

Q. 15. *How is the moment of inertia mathematically expressed ?*

Ans. If 'm' is the mass of the particle whose distance from the axis of rotation is r, then moment of inertia of the particle is measured by the quantity mr^2. Hence the moment of inertia of the whole body is expressed by Σmr^2.

Q. 16. *What is the significance of the symbol Σ (sigma) used in the expression Σmr^2 ?*

Ans. The symbol Σ (sigma) stands for summation of quantities. If the particles constituting the body have masses m_1, m_2, m_2,...m_n and their corresponding distances from the axis of rotation be r_1, r_2, r_3...r_n, then the moment of inertia of the body is equal to the sum of the moments of inertia of the constituent particles, that is

$$I=m_1r_1^2+m_2r_2^2+\ldots\ldots+m_nr_n^2$$

which can be (abbreviated as)

$$=\Sigma mr^2$$

Q. 17. *What is the significance of the words "Moment of Inertia" ?*

Ans. A rigid body is set rotating by a 'torque' which is simply "the moment of a couple". The body, in its turn, offers a passive resistance which is known as the 'Inertial reaction' of the body. This should also be a torque, since only a couple can counteract a couple. It is for this reason that this inertial reaction is designated as 'Moment of Inertia'.

Q. 18. *Can you give a clear cut definition of Moment of Inertia ?*

Ans. Yes. The moment of Inertia of a body about an axis is the sum of the products mr^2 taken for all the particles composing the body, where m is the mass of a particle, and r is its distance from the axis concerned.

Q. 19. *Is there any other way of defining moment of Inertia ?*

Ans. Yes. There is another way. If I be the moment of Inertia and $\frac{d^2\theta}{dt^2}$ be the angular acceleration of the body, then the deflecting couple 'C' is given by

$$C=I\,.\,\frac{d\,\theta}{dt^2}$$ from which we get the definition.

"The moment of Inertia of a body about an axis is equal to the moment of the couple required to produce unit angular acceleration in the body about the axis of rotation".

Q. 20. *The moment of Inertia in rotational motion is the counterpart of mass in translational motion. Now since the mass 'm' is constant, so the moment of inertia 'I' should also be a constant quantity. Is it not so ? Give one example.*

Ans. No ; it is not the case. The moment of inertia of a body is dependent on the manner in which the mass is distributed about the axis of rotation.

For instance, the moment of inertia of a cylinder about its own axis is $\frac{1}{2}Mr^2$, while the moment of inertia of the same cylinder about an axis passing through its middle-point and perpendicular to its axis is $M\left[\frac{l^2}{12}+\frac{r^2}{4}\right]$.

Q. 21. *When will the moment of Inertia of a body be minimum?*

Ans. The moment of inertia of a body will have a minimum value when it rotates about an axis which passes through its centre of gravity and which is normal to the smallest cross-section of the body.

Q. 22. *What do you mean by the term, "radius of gyration"?*

Ans. It is defined as the distance from the given axis of rotation to the point where the mass of the body may be supposed to be concentrated so that it has the same moment of inertia as it possesses about the given axis.

For instance, if the moment of inertia of the body be expressed in the form MK^2 where M is the mass of the body, then K is known as the radius of gyration.

Q. 23. *What is the radius of gyration of the regular body (Disc)?*

Ans. The disc is rotating about a normal axis passing through its centre. Under this circumstances, its moment of inertia is equal to $\frac{Mr^2}{2}$ so that the radius of gyration of this disc is $\frac{r}{\sqrt{2}}$.

Experiment 22 :

To find acceleration due to gravity with the help of a ball rolling down an inclined plane and by applying the correction for rotatory motion.

Apparatus :

Double inclined track, steel ball of about 2·5 cm diameter, wooden blocks (2 cm high), 2 weights of 1 kg each, stop-watch and clean cotton cloth.

Theory :

If the motion of a body involves translation as well as rotation then the sum of its potential energy, kinetic energy of translation and kinetic energy of rotation is conserved provided energy is not lost in overcoming friction.

The acceleration of a body rolling down an inclined plane is less than $g \sin \theta$, as its P.E. is converted partly into rotational K.E. and partly into translational K.E.

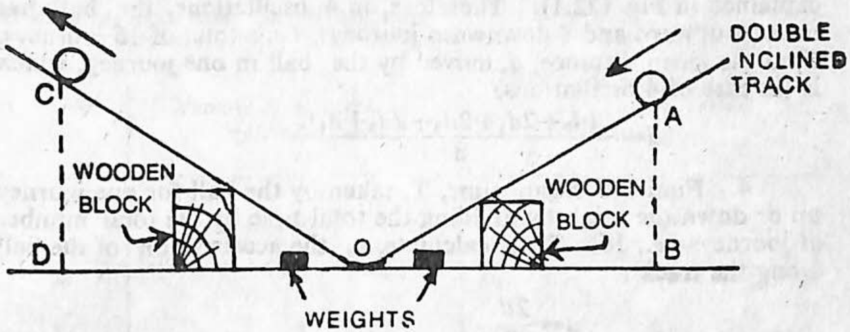


Fig. 22.1.

Procedure :

1. Put the inclined track on a table and keep the weights of 1 kg on the wings on either side of the middle of the track. Insert the wooden blocks under the ends of each track to incline the tracks (Fig. 22.1).

Release the steel ball from, say 50 cm mark on the left track and notice the mark upto which it reaches on the right track. Now release the ball from the same mark (50 cm) from the right track and notice the mark upto which it reaches the left track. If the inclinations of both the tracks to horizontal is same then the ball would rise to the same mark on both tracks on releasing. Suppose the ball rises to smaller length on the right track in the former trial, it indicates that the inclination of the left track is less than that of the right track. If this is the case then push the block under the left track a little towards the centre. Repeat the above procedure till the inclination of both the tracks to horizontal is the same.

While rolling, if the ball wobbles laterally during its journey from one track to the other, adjust the weights in the centre of the track laterally till there is no wobbling. The wobbling happens if the ball has to bend slightly to left or to right while moving from one track to the other, in the centre of the tracks, *i.e.*, if the two tracks are not in the same vertical plane.

2. Now hold the ball near the end on one of the arms. Note the distance d_0 from the centre at which you will release the ball. Then release it and simultaneously start the stop-watch. As the ball moves to and fro, note the successive distances d_1, d_2, d_3,..... from the centre upto which it ascends (after each complete oscillation) on the arm on which it started. Also keep a count of the number of oscillations that the ball has made. After, say, 4 oscillations of the ball, when you note the last distance d_4 upto which the ball ascends, simultaneously stop the watch. The last distance of ascent should not be less than 2/3 of the starting distance.

3. In the first oscillation, the ball makes two upward and two downward journeys and moves a total distance $2(d_1+d_1)$ as

explained in Fig. (22.1). Therefore, in 4 oscillations, the ball has made 8 upward and 8 downward journeys, *i.e.* a total of 16 journeys. Find the mean distance, d, moved by the ball in one journey, which is (in case of 4 oscillations)

$$d=\frac{(d_0+2d_1+2d_2+2d_3+d_4)}{8}$$

4. Find the mean time, T, taken by the ball for one journey up or down the track by dividing the total time by the total number of journeys *i e.*, 16. Then calculate, a, the acceleration of the ball along the track :

$$a=\frac{2d}{T^2}$$

5. Repeat the experiment ten times. In each case, find the acceleration. Then find the mean of the several values so obtained.

6. If r be the radius of the ball and k is its radius of gyration then acceleration is given by the relation

$$a=\frac{r^2}{(k^2+r^2)}\,g\sin\theta$$

This acceleration is less than $g\sin\theta$, the acceleration of a frictionless trolley moving down an inclined plane having same angle of inclination. It is due to the fact that the work done by the component of gravitational force along the direction of motion is partly converted into kinetic energy of translatory motion and partly into kinetic energy of rotatory motion of the ball.

For a solid sphere, $k^2=0{\cdot}4\,r^2$

Therefore, $g=\dfrac{a(k^2+r^2)}{r^2\sin\theta}=\dfrac{1{\cdot}4\,a}{\sin\theta}$

7. To determine the inclination sin θ, mark two points A and C on the two arms (Fig 22.1). Measure vertical heights AB and CD. Also measure distances of A and C from the centre, where height is zero. Find the two inclinations $\frac{AB}{AO}$ and $\frac{CD}{CO}$ with respect to the table and find their mean. If the table is not horizontal, this mean value gives the correct inclination of each arm of the apparatus with respect to the horizontal plane.

Since the heights AB and CD are quite small in this experiment, the best way to measure them is as follows :

Take a metallic block. Without disturbing the blocks which make the two arms inclined, find by trial and error where does this metallic block fit below each arm. The two positions of the block give the lines AB and CD. AO and CO are then measured by markings on the inclined tracks. AB and CD are equal to height of the metallic block which may be measured by a Vernier callipers. Find several pairs of values by using metallic blocks of different heights and find their mean.

Observations and Calculations :

S. No.	*Number of oscillations completed by ball*	*Distance of ascent on the arm on which it started (cm)*
1.	0	$d_0=$
2.	1	$d_1=$
3.	2	$d_2=$
4.	3	$d_3=$
5.	4	$d_4=$

Total time for 4 oscillations $=t=$ ....sec

Mean time for one oscillation

$$=T=\frac{t}{16}=\text{........sec}$$

Mean length of a single motion

$$=d=\frac{(d_0+2d_1+2d_2+2d_3+d_4)}{8}$$

$$=\text{........cm}$$

Acceleration of the ball

$$=a=\frac{2d}{T^2}=\text{........cms}^{-2}$$

S. No.	*Height of the block (cm)*	*Distance of upper edge from centre (cm)*	*Inclination*	*Mean Inclination*	*Inclination to horizontal sin θ*
Inclination of left arm					
Inclination of right arm					

Mean Inclination to the horizontal
$=\sin\theta=$........

Acceleration due to gravity
$$=g=\frac{1\cdot4\,a}{\sin\theta}=\ldots\ldots\ldots\text{cms}^{-2}$$

Result :

Acceleration due to gravity
at........= ....cms^{-2}

Value of acceleration due to
gravity at........from tables=........cms^{-2}
% error=

Precautions :

1. The track must be clean. Only then it will be possible to obtain 4 or 5 oscillations of the ball with the condition that the last distance of ascent is not less than about 2/3 of the starting distance. If the last distance of ascent is too small, the approximations involved in calculating mean time and mean length of one motion are not valid.

Use a small cotton swab moistened in an organic solvent (*e.g.*, carbon tetrachloride) for cleaning it.

2. The inclination of both the tracks to the horizontal is required to be the same.

3. The ball should not wobble laterally as it moves from one track to the other.

ORAL QUESTIONS

(Same as in Expt.16 and Expt. 21).

Experiment 23 :

To find the acceleration of a freely falling body using a ticker time and a tape.

Apparatus :

Ticker tape, timer, glass rod or tube or pulley for paper tape to pass over it, an object of mass about 100 g of iron (or brass) which can be easily suspended by the tap.

Theory :

Due to gravitational attraction of earth, any object when allowed to fall freely moves vertically down with the same acceleration irrespective of its size and material. This acceleration, known as acceleration due to gravity, is uniform for experiments on laboratory scale.

Procedure :

1. Set up the apparatus as explained in Experiment 18. Initially keep the body (100 g) hanging at rest by pressing the tape into the open portion of the ticker timer by the thumb. Start the ticker timer and release the body to let it fall to the ground.

2. Select 4 to 5 dots on a tape, roughly equally spaced in time, and find the velocity of the body at each of them as explained in Experiment 18. Also find the time at which the body reached each of these dots, taking the origin of time at the first of these.

3. Plot a velocity-time graph for the motion of the body.

4. Find the slope of the graph to determine its acceleration.

Observations and Calculations :

Time period of the timer=......sec

S. No. of dot selected	*Time when the body reached the selected dot*	*Velocity* $V=\frac{s}{2nT}$ (*in* ms^{-1})

Slop of the graph$=\frac{dV}{dT}$

$=......ms^{-2}$

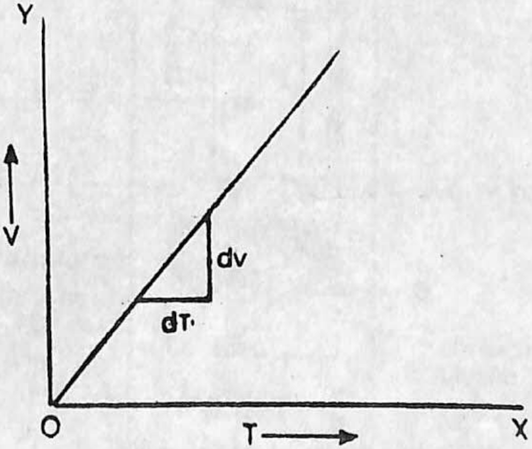


Fig. 23·1.

Result :

The acceleration of the given freely falling body$=......ms^{-2}$

Precautions :

1. Leave a few dots in the beginning of the motion since they would be too close together and their analysis may be difficult.

2. Notice the dot just before the object hits the ground. Discard the dots if any marked on the tape after the object touches the ground.

3. Clamp the ticker-timer near the edge of the table properly so that the paper tape after passing over the glass rod hangs vertically.

4. There should not be any drag on the paper tape as the body falls freely under gravity.

ORAL QUESTIONS

(Same as in Experiment 6)

SECTION E

Experiment 24 :

To study a Fortin's barometer and to measure the atmospheric pressure at a room temperature at a place.

Apparatus :

(1) Fortin's barometer (2) A thermometer (3) Plumb line.

Theory :

(*i*) Pressure $= H_o dg$ dynes/cm^2 where
H_o = corrected barometric height
d = density of mercury = 13.6 gm/cm^3
and g = Acceleration due to gravity.

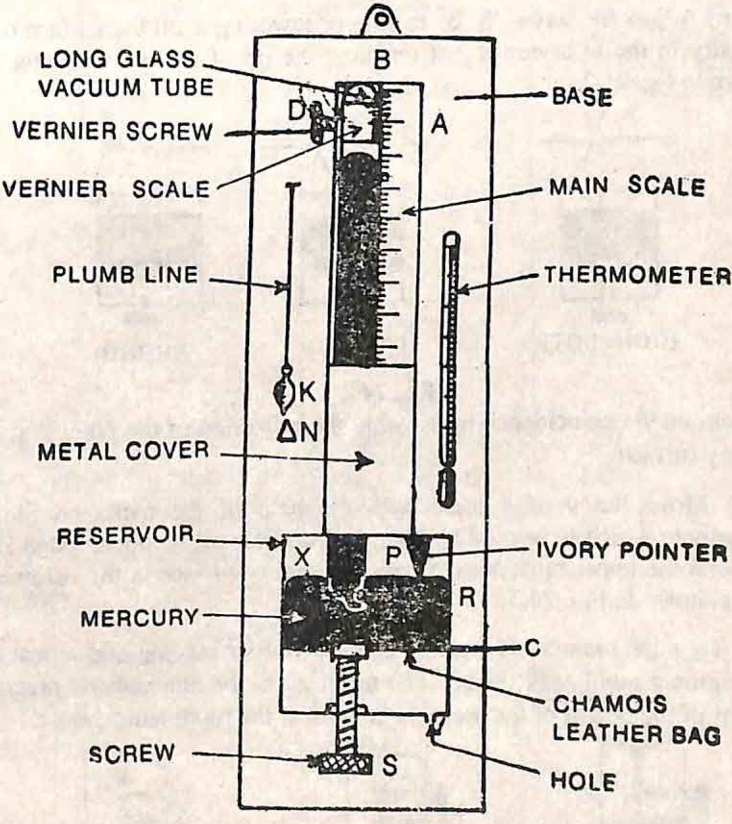


Fig 24·1. Fortin's Barometer

(*ii*) $Ho = H_t\,(1-(\gamma_r-\alpha)t]$
where H_t = Barometer reading at t°C.

H_o = Barometer height i.e., the height of mercury column giving the atmospheric pressure at 0°C.
α = Coefficient of linear expansion of the material of main scale.
γ_r = Coefficient of real cubical expansion of mercury.
t = Room temperature

Now $\alpha = 0.000018$ for brass
$\gamma_r = 0.00018$ for Mercury

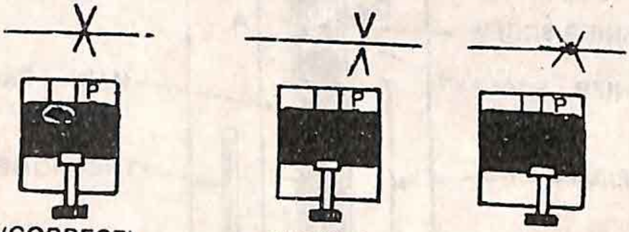
$\therefore$ $H_o = H_t\,[1 - 0.000162t]$

Procedure :

(i) Find the vernier constant of the vernier.

(ii) Adjust the base till the points K and N of the plumb line are just in the same vertical line.

(iii) Adjust the screw 'S' by raising or lowering it till the surface of the mercury in the reservoir is just touching the tip of the ivory pointer P as shown in fig. 24.2.



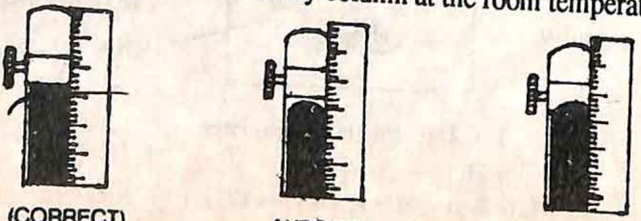
(CORRECT) (WRONG) (WRONG)

Fig. 24·2.

Examine the coincidence by viewing the reflection of the point P at the mercury surface.

(iv) Move the vernier scale with the help of the rack and pinion arrangement which is worked by means of a screw till the lower edge is in level with the upper most convex part of mercury surface in the barometer tube as shown in Fig. 24.3.

(v) Take the main scale reading and the vernier reading and repeat the observation a number of times. The mean gives the atmospheric pressure in terms of the height of the mercury column at the room temperature.



(CORRECT) (WRONG) (WRONG)

Fig. 24.3.

(vi) Note the room temperature with the help of the thermometer fitted with the barometer tube. Apply the correction and find the value of H_o i.e. the barometer height at 0°C.

Observations and Calculations :

No. of divisions per cm on the Main Scale $= m =$ ....

No of divisions on the vernier scale $= n =$ ....

$$\therefore \text{ Vernier constant} = \frac{1}{m} \times \frac{1}{n} = \ldots\ldots \text{cm}$$

(a) Atmospheric Pressure

Room temperature $(t) =$ ......°C

S. No.	Fortin's Barometer Reading			H_t=Height of Hg column at t°C =(1)+(2) in cm
	Main Scale Reading (1) cm	*Vernier division coinciding*	*Vernier Value (1) in cm*	
1.				
2.				
3.				
4.				

Mean Height H_t of Mercury column = ...cm

Now $H_o = H_t\,[1 - 0{\cdot}000162\ t] =$ ....cm.

$\therefore$ Atmospheric Pressure at 0°C $= P_o = H_o\ dg =$ ....dynes/cm^2

Result :

Atmospheric Pressure is

= ....cm of Hg column at 0°C

= ....dynes/cm^2 at 0°C

= ....N/m^2

at.........(Place of experiment)

on.........(Date of experiment)

Precautions :

1 The barometer tube must be vertical.

2. The surface of the mercury in the bag should just touch the tip of the pointer in the bag.

3. The lower edge of the vernier scale should just appear to touch the upper-most portion of the mercury surface.

4. The eye must be at the same level as the vernier scale.

Sources of Error

1. The space above mercury is not complete vacuum but contains vapours of mercury which exert a very small pressure.

2. Any change in the room temperature changes the density of mercury which in turn affects the observation.

Exercise

Q. 1. *To find the height of the water barometer in the laboratory ?*

Hint : Height of the water barometer $= H_0 \times 13{\cdot}6$ cm

ORAL QUESTIONS

Q. 1. *What is Atmospheric Pressure ?*

Ans. The air has its own weight and so exerts a pressure. The amount of this force per unit area exerted by the atmosphere is called the Atmospheric Pressure. It is measured in terms of mercury column that this pressure can support. Normal atmospheric pressure is 76 cm of Hg.

Q. 2. *What is the unit of Pressure ?*

Ans. Its unit is dynes/cm^2 in C.G.S. system and newton/metre^2 in the S.I. system.

Q. 3. *Why is the atmospheric pressure expressed in terms of length ?*

Ans. As the pressure $= h\ d\ g$ and as 'd' and 'g' are constant, therefore, pressure $\propto h$. The height is, therefore, taken as a convenient measure.

Q. 4. *Why do you use mercury in a barometer ? Can we use water ?*

Ans. We use mercury because it is the heaviest liquid and hence the length of the barometer is conveniently small, about 76 cm.

We can use water also but since mercury is 13·6 times heavier than water, the height of water barometer will be $76 \times 13{\cdot}6 = 10{\cdot}3$ m approximately which will be very cumbersome. Besides this, mercury exerts very small vapour pressure.

Q 5 *Why should the surface of the mercury be adjusted to touch "Ivory Point" in a Fortin's Barometer ?*

Ans. Ivory point represents the zero of the vertical main scale and the surface of mercury has to be adjusted to zero level to take the correct reading.

Q. 6. *Is the pressure the same in every direction ?*

Ans. Yes : the pressure in different directions at the same level is the same.

Q. 7. *State Pascal's Law.*

Ans. Pressure applied anywhere on a confined liquid is transmitted equally in all directions.

Q. 8. *If the Barometer tube were wider, will there be any change in the height of mercury column ?*

Ans. No ; the shape of the tube has no effect upon the vertical height of the mercury column supported in the tube.

Q. 9. *If the Barometer tube were inclined, will there be any change in the height of mercury column ?*

Ans. It is the vertical height of mercury column which is a measure of atmospheric pressure. If the tube is inclined a little, the space above the free surface of mercury decreases, but its vertical height remains the same.

Q. 10. *Do you know any barometer which does not contain liquid ?*

Ans. Yes ; Aneroid Barometer—a more convenient and portable form of barometer whose reading is not affected by any change in position.

Q. 11. *Which barometer will you carry in an aeroplane to know its height and why ?*

Ans. Aneroid Barometer. The Aneroid Barometer when used to find height attained by an aeroplance is known as *Altimeter.*

It is very compact and portable and is very sensitive to changes in atmospheric pressure.

Q. 12. *How do you forecast weather changes with a barometer ?*

Ans. (*i*) A falling barometer indicates the coming of rain and storm because the moist air is lighter than dry air, its density being 0·62 times as much as that of air.

(*ii*) a steady barometer indicates settled weather, and

(*iii*) a rising barometer indicates fair and dry weather.

Q. 13. *What do you understand by the statement that the atmosphere pressure is 76 cm of mercury ?*

Ans. It is the pressure exerted by a column of mercury 76 cm in height at 0°C at sea-level and at latitude 45°. This is known as the pressure of *one atmosphere* and is taken to be the standard for comparison.

The standard atmospheric pressure
$= 76 \times 13{\cdot}6 \times 981$ dynes/cm^2
or
The Normal Atmospheric Pressure
$= 1{\cdot}013 \times 10^6$ dynes/cm^2
$= 1{\cdot}013 \times 10^5$ N/m^2

Q. 14. *Does the atmospheric pressure change as one moves up in the atmosphere ? If so, in what manner ?*

Ans. As we go higher into the atmosphere, the pressure decreases For some altitudes, the fall in the atmospheric pressure is about 0·25 cm for every 270 metres ascent Thus a barometer, which reads 75 cm at sea level will read 74·75 cm at 270 m above sea level. After a few hundred metres, however, the fall in pressure is not regular

Q. 15. *What will happen if a hole is drilled at the top of the barometer tube.*

Ans. Mercury column will not stay in the barometer tube

Experiment 25.

To measure atmospheric pressure and study the variation of volume of a sample of air with its pressure at constant temperature. Also use the study to find the atmospheric pressure.

OR

To verify Boyle's Law for a gas and use it to find the atmospheric pressure.

Apparatus :

Boyle's Law apparatus, Fortin's Barometer, Thermometer plumb line and Set squares.

Theory :

According to Boyle's Law, "Temperature remaining constant, the pressure of a given mass of a gas is inversely proportional to its volume".

$$P \propto \frac{1}{V}$$

or PV = constant (provided the temperature T is constant) ;

so if a graph is drawn between P and V, it will be found to be a *rectangular hyperbola* because PV = constt. is the equation for it but the graph between P and $\frac{1}{V}$ will be a *straight line.*

But $V = a\,l$ where l is the length of the air column and 'a' is the area of cross-section of the tube.

$$\therefore \quad P\,a.l = \text{constt.}$$

As 'a' is constt ; therefore, $Pl = \text{constt.}$

or $\quad P_1l_1 = P_2l_2 = P_3l_3 = \ldots\ldots = \text{constt.}$

or $$P \propto \frac{1}{l}$$

$\therefore$ Graph between P and $\frac{1}{l}$ will be a straight line.

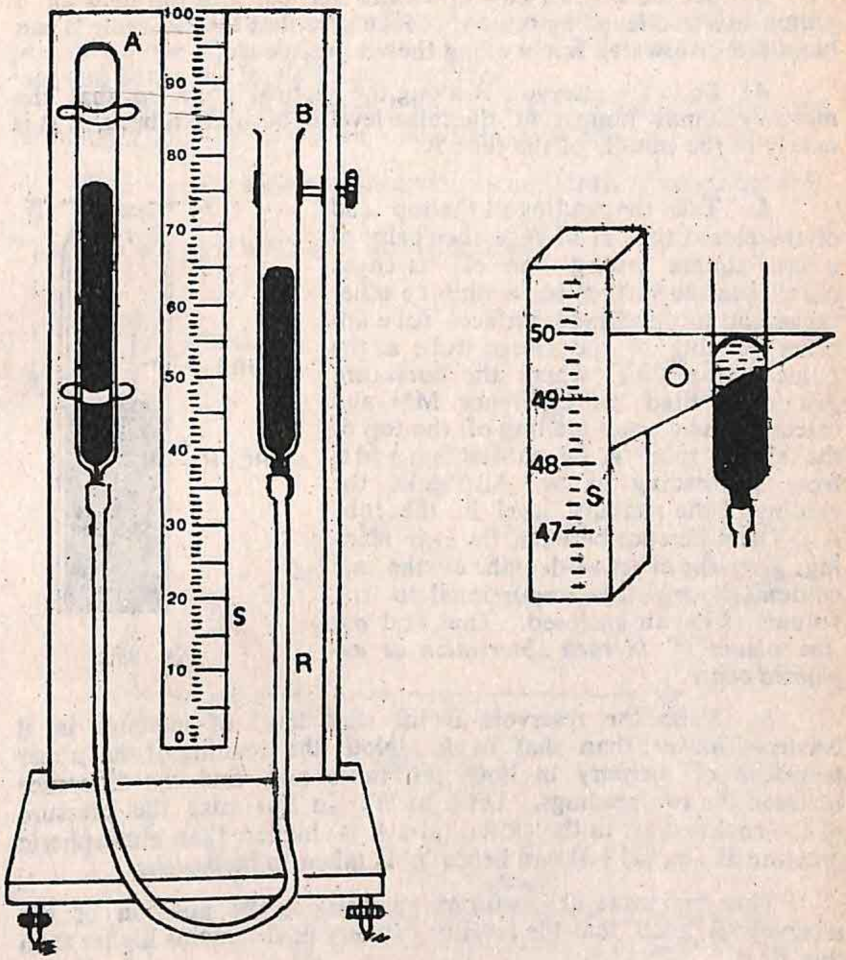


Fig. 25.1 Boyle's Law Apparatus

Procedure :

1. Find the vernier constant of Fortin's Barometer. With the help of the screw provided at the bottom end of the barometer, adjust the mercury level in the cistern till the mercury level touches the ivory head in the cistern. Hold eye in front of the mercury level in the tube. Slide the vernier till zero of the vernier just touches the mercury level. Read the mercury level on the scale. It gives the atmospheric pressure (H).

2. Note the room temperature by the thermometer provided by the barometer. If not provided, use another thermometer.

3. Set the Boyle's Law apparatus vertical with the help of a plumb line and levelling screws. Make sure that the reservoir B can be raised or lowered freely along the vertical board.

4. Slide the reservoir B along the vertical scale so that the mercury stands almost at the same level in both the tubes and it is nearly in the middle of the tube A.

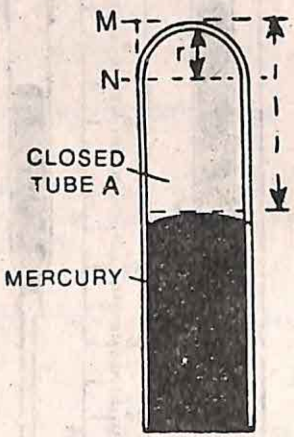


Fig. 25·2.

5. Take the reading at the top end of the closed tube at M with the help of a set square setting one of its edges parallel to the vertical scale and the other tangential to the curved surface. Take another reading of the closed tube at the point N (Fig 25 2) where the curvature just ends. Find the difference MN and calculate the correct reading of the top of the closed tube A by subtracting $\frac{1}{3}$ MN from the reading at M. Also take the reading of the mercury level in the tube A. The difference between the two readings gives the effective length of the air column (l) which is proportional to the volume of the air enclosed. Thus find *out the volume 'V' in each observation as explained above.*

6. Raise the reservoir B till the level of mercury in B becomes *higher* than that in A. Note the reading of the upper meniscus of mercury in both the tubes and find the difference between the two readings. Let it be 'h'. In this case the pressure of the enclosed air in the closed tube A is higher than atmospheric pressure H *i.e.*, $(H+h)$ and hence 'h' is taken to be *positive.*

Take two more observations by changing the position of the reservoir B such that the level of mercury in B remains higher than that in A.

Now lower the reservoir such that the level of mercury in B is *lower* than that in A. Again note the reading of the mercury levels in both the tubes A and B as before. Take the difference and let it be 'h' In this case the pressure of the enclosed air in the tube A is lower than the atmospheric pressure H *i.e.*, $(H-h)$ and hence 'h' is taken to be negative. Take *three* observations by keeping the level of mercury in the tube B lower than that in A.

Thus in all take *six* observations.

7. Take the barometer reading again at the end of the experiment. Also note the room temperature at the end of the experiment.

8. Multiply the pressure $P=(H\pm h)$ of the gas with its respective volume 'V' and show that the product PV=constant in each case and hence the Boyle's Law is verified.

9. Plot a graph between P and V, taking P along X-axis and 'V' along Y-axis which will be a parabola (Fig. 25.4).

Find $\frac{1}{V}$ for different observations and draw a graph between P and $\frac{1}{V}$ which is a straight line (Fig. 25.3).

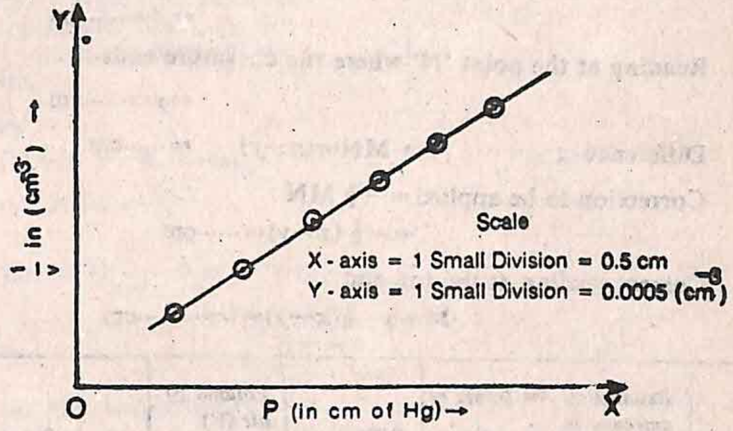


Fig. 25·3.

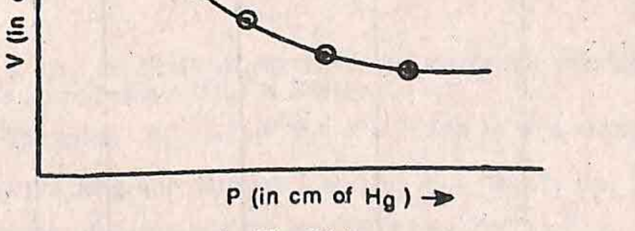


Fig. 25·4.

Observations and Calculations

Room temperature at the beginning of the experiment=......°C

Room temperature at the end of the experiment=......°C

Barometer reading in the beginning$=H_1=$......cm of Hg

Barometer reading at the end $=H_2=$......cm of Hg

Mean barometric reading $=H=\frac{H_1+H_2}{2}$

=.........cm of Hg

Reading at the top end 'M' of the closed tube

$=x=$......cm

Reading at the point 'N' where the curvature ends

$=y=$......cm

Difference $=MN=(x-y)$ =......cm

Correction to be applied$=-\frac{1}{3}$ MN

$=-\frac{1}{3}(x-y)=$......cm

Correct reading at the top end

$M=x-\frac{1}{3}(x-y)=l=$.........cm

No. of obs.	*Reading at the levels of mercury in*		*Difference $I\sim II=h$ (cm)*	*Volume of air (V) $=I-II$ in cm ($V\propto$ effective length)*	$\frac{1}{V}$	*Pressure of gas $P=H\pm h$ (cm of Hg)*
	tube B (I) (cm)	*tube A (II) (cm)*				
1.						
2.						
3.						
4.						
5.						
6.						
7.						

Atmospheric Pressure :

(*a*) **Graphically** ;

Produce the straight line obtained from the graph between difference '*h*' in the mercury levels of the two tubes A and B (Fig 25.1) and $\frac{I}{V}$ so as to meet the Y-axis at 'C' as in Fig. 25.5. Note the readings of the pressure OC at this point which is atmospheric pressure.

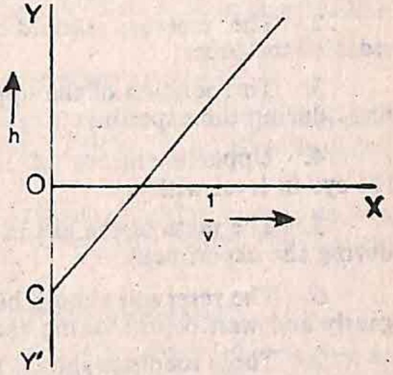


Fig. 25·5.

Theory :

$$(H+h)\ V=K\ (\text{constt.})$$

$$\text{or}\quad H+h=\frac{K}{V}$$

$$\text{But at C,}\ \frac{I}{V}=0\ \text{or}\ (H+h)=0$$

$$\text{or}\qquad \boxed{H=-h}$$

(*b*) **Mathematically** :

The atmospheric pressure 'H' can easily be determined from any two different observations. For differences of mercury levels 'h_1' and 'h_2' let the corresponding gas volumes be V_1 and V_2. Then from the relation.

$(H \pm h_1)\ V_1=(H \pm h_2)\ V_2$; H can be calculated. The most important precaution for this experiment is that V_1 and V_2 should differ by at least 3 cm. The smaller the difference, the greater the percentage error.

Result :

(*i*) Within the limits of experimental errors, the products PV are seen to be constant. This is Boyle's Law.

(*ii*) The graph between P and V is found to be a rectangular hyperbola and the graph between P and $\frac{1}{V}$ is a straight line. These two graphs also demonstrate the verification of Boyle's Law.

(*iii*) The atmospheric pressure on......(date) at......(time) and at......°C=...... cm of Hg (By graph)

=......cm of Hg (By calculation)

Precautions :

1. The apparatus should be vertical.

2. The mercury should be clean and should not stick to the sides of the tubes.

3. The position of the upper end of the tube A should remain fixed during the experiment.

4. Upper meniscus of the mercury should be read keeping the eye in level with it.

5. The mass of the gas in the tube A should not be changed during the experiment.

6. The reservoir should be raised or lowered very slowly and gently and wait before taking each reading.

7. Three readings should be taken for pressure greater than the atmospheric and three less than the atmospheric pressure.

8. Readings should be taken with the help of the set squares.

9. Readings of the atmospheric pressure and the room temperature should be taken before and after the experiment.

10. The correction for the curved part of the tube in the determination of the length of the air column should be applied.

Sources of Error :

1. The enclosed air may not be dry.

2. The room temperature and barometric height may change during the experiment.

3. The mercury used is generally not as pure as in barometer and so there is a difference in their densities.

4. The closed end may not be uniform.

ORAL QUESTIONS

Q. 1. *What is Boyle's Law ?*

Ans. Temperature remaining constant, the pressure of a certain mass of a gas is inversely proportional to its volume, *i.e.,*

$$P \propto \frac{1}{V} \text{ or } PV = \text{constt.}$$

Q. 2. *What are the limitations of Boyle's Law ?*

Ans. Actually gases show deviation from this law. Permanent gases, *e.g.,* air, N_2, O_2, H_2, etc., obey it fairly well under ordinary pressure and temperature.

It does not hold good at very high pressure and at very low temperature. It is not true for saturated vapours.

Q. 3. *What is a perfect (or ideal) gas ? Name some of them.*

Ans. A perfect gas is that which obeys Boyle's Law and Charle's Law. In fact no gas is perfect but permanent gases like O_2, H_2, N_3, etc., are very nearly so.

Q. 4. *What is the difference between gas and vapour ?*

Ans. A substance below its critical temperature is called a vapour, whereas a substance above its critical temperature is called a gas.

Vapour can be liquified by increasing pressure alone, while a gas cannot be liquified and has to be cooled below the critical temperature before it can be liquified.

Q. 5. *How do you keep the temperature constant ?*

Ans. By waiting for a short time after every observation so that the heat developed during compression may dissipate out.

Q. 6. *Should the change be isothermal or adiabatic in Boyle's Law ?*

Ans. Isothermal.

Q. 7. *How do you explain the fact that a small amount of gas in the tube sometimes exerts a greater pressure than even the whole column of the atmosphere ?*

Ans. During compression, the bombardment of the molecules of the enclosed gas against the sides of the tube increases and hence the pressure as the pressure is proportional to this bombardment and not to the mass of the gas.

Q. 8. *What is the difference between the following relations :*

(a) $PV=\text{constt.}$ and (b) $PV=RT$.

Ans. The first relation is Boyle's Law and the second relation is the gas equation obtained by combination of Boyle's Law and Charle's Law.

Q. 9. *What is the use of drying the gas ?*

Ans. Boyle's Law does not hold good accurately for moist gases. Application of pressure will condense some of the water vapour into water and hence the gas should be dry.

Q. 10. *Which graph will you prefer and why ?*

(a) P and V, (b) P and $\frac{1}{V}$, (c) V and $\frac{1}{P}$.

Ans. P and $\frac{1}{V}$ or V and $\frac{1}{P}$ graphs, as these are straight lines and errors are easily accounted for.

Q. 11 *Has the diameter of either the gas tube or the reservoir anything to do with the pressure to which the gas is subjected ?*

Ans. There will be no effect in either case, simply the quantity of mercury required will be larger in the case of a wider tube.

Q. 12. *If in the Boyle's Law apparatus, the tube A is not graduated in c.c., how will you measure the volume in terms of length of the tube A above the level of mercury in it ?*

Ans. The upper part of the tube A (Fig. 25.2) is curved and as a result of it, a correction has to be applied for it. This is calculated as follows :

Volume of the curved portion = Volume of the hemisphere of radius 'r'.

$$= \frac{1}{2} \times \frac{4}{3} \pi r^3 = \frac{2}{3} \pi r^3.$$

If the curved portion had been flat, it would have formed a cylinder of length 'r' and also of radius 'r'. Then the volume of this cylinder $= \pi r^2 \times r = \pi r^3$

$\therefore$ Error in volume $= \pi r^3 - \frac{2}{3} \pi r^3 = \frac{1}{3} \pi r^3$

$\therefore$ The resulting error in length $= \frac{\text{Volume}}{\text{area}} = \frac{1}{3} \frac{\pi r^3}{\pi r^2} = \frac{1}{3} r$

Correction in length $= \frac{1}{3} r = -\frac{1}{3}$ MN

Thus in order to obtain correct length (or volume as volume $\propto$ effective length), $\frac{1}{3} r$ should be subtracted from the observed length.

Experiment 26:

To measure the air pressure inside a balloon by using an open-ended water manometer.

Apparatus :

A rubber balloon and an open-ended water manometer.

Theory :

A barometer measures atmospheric pressure, but in science we often need to measure pressures other than that of the atmosphere. Instruments used to measure these are known as *pressure gauges* or *manometers.*

A water manometer consists of a U-tube containing coloured water (Fig. 26.1).

When we blow a balloon, it inflates in all directions. This shows that air inside it exerts pressure on all the sides.

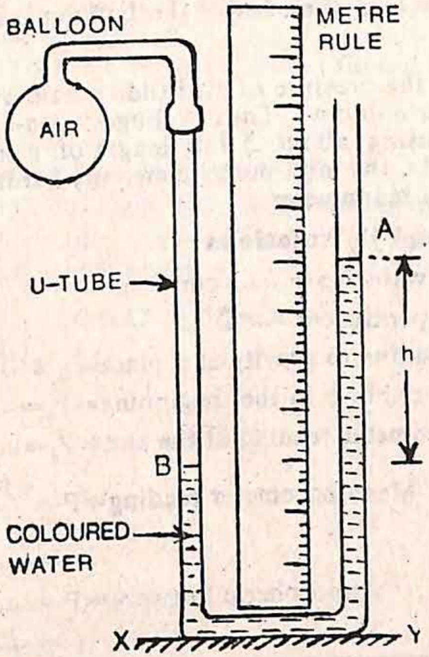


Fig. 26.1 (Water Manometer)

The pressure of air inside the balloon is more than the atmospheric pressure by an amount equal to $h\rho g$ where h is the difference in levels of water in the two limbs of the water manometer (Fig. 26.1) ; ρ is the density of water and g is acceleration due to gravity at a place.

Procedure :

1. Blow a rubber balloon. Connect it to one of the limbs of the manometer containing coloured water as shown in Fig. 26.1.

The water level in this limb of the manometer will be depressed due to the pressure of air inside the balloon. The level of water in the other limb will rise.

2. When the water in the manometer is steady, measure the heights of water meniscus at A and B above a flat glass plate (XY) placed on the table. The difference in levels, h, of water in the two limbs is measured *i.e.*, AB$=h$. AB$=h$ is also called "*head of water*".

3. The pressure of air in the balloon is, then, more than the atmospheric pressure by an amount equal to $h\rho g$ where ρ is the density of water and g is acceleration due to gravity.

4. Find the atmospheric pressure P at a place using Fortin's barometer as explained in Experiment 24.

5. The pressure of air inside the balloon is equal to $P+h\rho g$.

Note :

Sometimes the pressure of air inside a balloon can be as much as 1·5 m of water column Thus a huge open-ended manometer should be made using about 3·0 m length of a transparent plastic tube. Alternatively, the multipurpose mercury barometer can be used as an open-ended manometer.

Observations and Calculations :

Density of water$=\rho=$......g cm^{-3}

Room temperature=......°C

Acceleration due to gravity at a place$=g=$......cm/s^2

Barometer reading in the beginning$=P_1=$......cm of Hg

Barometer reading at the end$=P_2=$......cm of Hg

Mean barometer reading$=P=\dfrac{P_1+P_2}{2}$

=......cm of Hg

∴ Atmospheric Pressure$=P=$......cm of Hg

=......dyne/cm^2

Reading at the level of water in		*Difference in levels AB=h (in cm)*	*h ρ g (dyne/cm²)*	*Pressure of air inside the balloon =P+h ρ g (dyne/cm²)*
Limb A	*Limb B*			

Result :

The pressure of air inside a balloon=......dyne/cm^2

=......N/m^2

Precautions :

1. The apparatus should be vertical.
2. The water should not stick to the sides of the limbs.
3. The lower meniscus of water should be read keeping the eye in level with it.
4. Readings should be taken with the help of the set squares.

5. Readings of the atmospheric pressure should be taken before and after the experiment.

6. The mass of air in the balloon should not be changed during the experiment

Sources of Error :

1. The air in the balloon may not be dry.
2. The barometer height may change during the experiment.
3. The water droplets may remain sticking to the limbs of the manometer during the experiment.

ORAL QUESTIONS

(Same as in Expt. 24)

SECTION F

Experiment 27 :

(*a*) *To estimate the size of a molecule of an oil.*
(*b*) *To estimate the number of oil molecules in 1 gram-molecule.*

Apparatus :

Burette, small beaker or conical flask, glass bottom tray (30″ × 30″) with graph paper attached to the underside, lycopodium powder, small

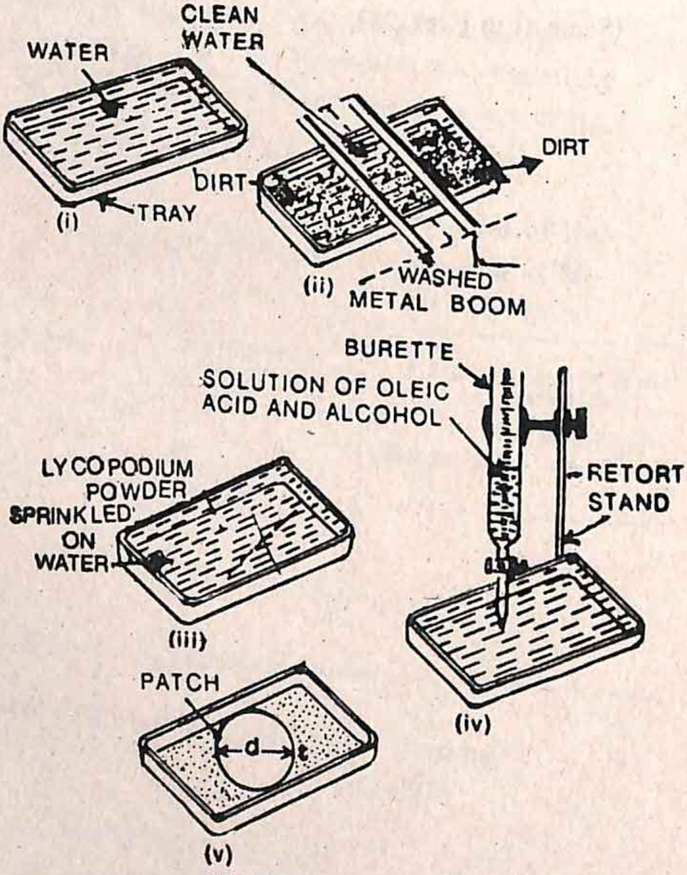


Fig. 27.1.

measuring cylinder or pipette or dropper, of solution concentration 1 in 100 by volume of oleic acid in one litre of Benzene, two waxed metal booms, cheese cloth cover, retort stand, meter rule, bucket, four rubber wedges, microscope, magnifying glass etc.

Theory :

According to Lord Rayleigh's work and of the assumption, or guess, that he made—*the oil spreads until it is a sheet one molecule thick.*

Allow one drop of given solution of oleic acid to fall on the water in a tray. It spreads over the surface equally in all directions forming a circular thin film of thickness (t) and diameter (d). Estimate the area of the patch of clear liquid formed. If the patch is roughly circular, we can measure its diameter. Hence deduce the thickness of the layer of oleic acid. If we assure that the layer is one molecule thick, this gives one dimensions of the oleic acid molecule. (This will happen if the surface of water is grease free).
Area of the patch × thickness of the
layer of oleic acid = volume (V) of the oleic acid drop
or
(size of the oleic
acid molecule)

$\therefore$ size of the oleic acid molecule

$$=\frac{\text{volume of the oleic acid drop (V)}}{\text{area of the patch}}$$

or

$$t=\frac{V}{\frac{\pi d^2}{4}}=\frac{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3}{\frac{\pi d^2}{4}}=\frac{2}{3}\,\frac{D^3}{d^2}$$

Value of t depends very much on the accuracy of the measurement of diameter (D) of oil drops.

Procedure :

1. Shake well the given solution of oleic acid. Take 1 c.c. of it and dilute to 100 c.c. with 1 litre of benzene.

2. In order to estimate the diameter of the oleic acid drop, fill the dropper (with narrow bore) with oleic acid solution. Now empty the dropper drop by drop in the burette till the volume emptied is one c.c. and also count the number of drops. Find out the volume (V) of a drop that dropper gives.

Alternatively, we can also find the volume of the oleic acid drop as follows :

Take a wooden piece of about 4″ and 2½″ and fix a nail on it. Make a loop of thin wire as shown.

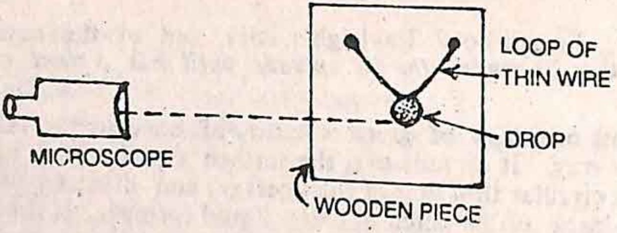


Fig. 27.2

A glass rod is dipped in the oleic acid solution and touched at the lower part of the wire so that the drop of suitable size is formed and its size adjusted by dragging more drops close to it or away from it.

Now set the travelling microscope for the drop and measure its diameter (D) upto the accuracy of 0·001 cm and its volume is given by $V=\frac{4}{3}\pi\left(\frac{D}{2}\right)^3$

3. Place the given glass bottom tray (30″×30″ as used in photography) with graph paper attached to the underside on the bench. The tray is cleaned with clear cotton and detergent powder and rinsed with water a number of times without touching any of its sides with fingers so that it is free from grease etc. Fill the tray partially with clean tap-water and level it carefully by using rubber wedges. Now fill the tray to *over-brimming* with further levelling (Fig. 27.1).

The tray should be a *large*, enamelled metal *tea* tray, painted black. The tray should not be of porous wood or plastic that might retain oil. Tray must have a *broad flat* and *rounded rim*.

4. Clean the water surface by moving the two *waxed metal booms slowly from the middle to the two ends of the tray [Fig 27.1 *ii*)]. The booms should be left near the end of the tray.

5. When the water in the tray has become quite steady sprinkle the surface with lycopodium powder. [Fig. 27.1 (*iii*)].

6. Arrange the burette so that its jet is just above the surface of the water in the tray [Fig. 27.1 (iv).]

7. Allow *one* drop of the solution to fall on the water, *in the centre* of the tray.

---

*Melt paraffin wax in a can and carefully paint the booms with a layer of molten paraffin wax with a soft paint brush. If any difficulty in waxing is experienced, it is almost certainly due to not having the wax hot enough.

8. Using the graph paper estimate the area of the **patch of clear liquid formed [Fig. 27.1 (*v*)]. If the patch is rough circular then use metre rod to measure the maximum diameter of the patch produced in two directions.

9. Deduce the thickness of the layer of oleic acid. If we assume that the layer is one molecule thick, this gives one dimension of the oleic acid molecule.

10. Knowing the density of oleic acid and assuming that its molecules are of roughly *cubic* shape, estimate the volume of one molecule and hence its mass. Knowing molecular weight of oleic acid, estimate the number of molecules in one gram-molecule.

Observations and Calculations :

Volume of one drop of oleic acid $= V = \ldots\ldots \text{cm}^3$

Area of the patch of clear liquid $= \ldots\ldots \text{cm}^2$
(using graph paper)

Or

For Roughly Circular Patch

Maximum diameter of the patch (roughly circular) on one side $= d_1 = \ldots\ldots \text{cm}$

Maximum diameter of the patch on the other side $= d_2 = \ldots\ldots \text{cm}$

Mean maximum diameter of the patch

$$= d = \frac{d_1 + d_2}{2} = \ldots\ldots \text{cm}$$

Area of the patch $\simeq d \times d = \ldots\ldots \text{cm}^2$

If we assume that the layer is one molecule thick, then,

Let thickness of the layer of the oleic acid $= t$ cm (*i.e.*, the size of molecule)

Area of the patch $\times\, t =$ volume of oleic acid drop (V)

$$\therefore \qquad t = \frac{V}{\text{Area of the patch}} = \ldots\ldots \text{cm}$$

$$= \ldots\ldots \text{Å}$$

Thus the size of the oleic acid molecule $= \ldots\ldots$ Å

**With some water supplies, the patch contracts to a smaller size soon after it is formed. This is probably due to water-softening agents attacking the oleic acid, though this is not certain. Whatever the cause of such a contraction, we believe the proper measurement to take is the initial maximum diameter.

Density of oleic acid$=0{\cdot}90\pm0{\cdot}01$ gm cm^{-3}. Assuming that the molecules of oleic acid solution are of roughly *cubic* shape,

The *volume* of *one* molecule $\simeq t^3=\ldots\ldots$cm^3

$\therefore$ Mass of one molecule $=M=t^3\times(0{\cdot}90\pm0{\cdot}01)$

$=\ldots\ldots$g

Molecular weight of oleic acid$=282$

$\therefore$ No. of molecules in one gram-molecule

$=\ldots\ldots$

Result :

Roughly, the size of oleic acid molecule$=\ldots\ldots$Å

Roughly, the number of molecules in 1 gram—molecule

of oleic acid$=\ldots\ldots$

Note. Experimentally the value of diameter (*d*) of circular patch$=25$ cm ; diameter of the oleic acid (D)$=0{\cdot}095$ cm and the molecular size $t=68$A°.

Precautions and Sources of Error :

1. The tray should be water-proof.

2. The water surface should be swept clean just before each student places his acid drop on it, by moving two water-proof waxed metal booms out from the centre to the ends.

3. The measurements are obviously rough but worth having. Treat the oil drop as a cube, or width equal to the diameter of the sphere (Fig. 27.4). Treat the oil patch as a square instead of a circle (Fig. 27.3) a square of width equal to the diameter of the circle. That will yield a smaller estimate of molecule—height, two-thirds of the proper result—but never mind, simplicity is best here.

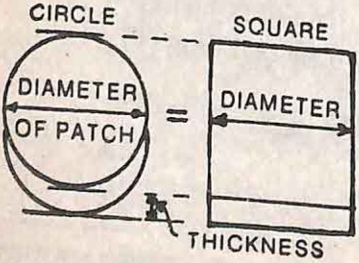


Fig. 27·3.

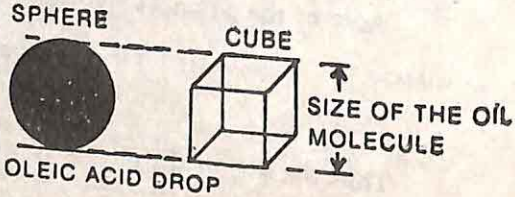


Fig. 27·4.

4. The liquid should be dropped only drop by drop.

ORAL QUESTIONS

Q. 1. *What is the aim of this experiment ?*

Ans. To estimate the size of the molecule.

Q. 2. *What is a molecule ? What is molecular size ?*

Ans. Molecule is the smallest part of matter which can exist independently and continues to possess all the properties of the matter.

Molecular size means molecular diameter.

Q. 3. *Why should the oleic acid be diluted ?*

Ans. This is done to take a small trace of the oil even by trapping larger volume of the solution.

Q. 4. *What is the purpose of lycopodium powder in this experiment ?*

Ans. To have a circular patch of the clear liquid.

Q. 5. *What happens to benzene or alcohol in the film ?*

Ans. The benzene or alcohol contained in the drops evaporates partly and its remaining part gets dissolved in water, leaving behind only the oil molecules to spread over the surface of water.

Q 6. *What type of substances whose molecular size you estimate are suitable for this experiment ?*

Ans. Those which spread into a circular film of *mono-molecular layer*.

Q. 7. *What is a mono-molecular layer ?*

Ans. A layer whose thickness is equal to the diameter of a molecule.

Q. 8. *As the molecules of oils and fats are not spherical, is the thickness of the mono-molecular layer equal to the smaller dimensions of the molecule.*

Ans. No : it is equal to the largest dimension of the diameter.

Experiment 28.

To measure surface tension of water by capillary rise method

Apparatus :

Capillary tubes of uniform bore and different diameters (ranging from 0·5 to 1·5 mm) ; a glass strip, a thin rubber tube band, a needle, a glass vessel, a clamp stand, a travelling microscope ; a thermometer and an adjustable stand.

Theory :

When a capillary tube open at both ends is dipped vertically in a liquid, the surface of the liquid inside the tube is generally *curved*. If the liquid wets the tube as in the case of water, the

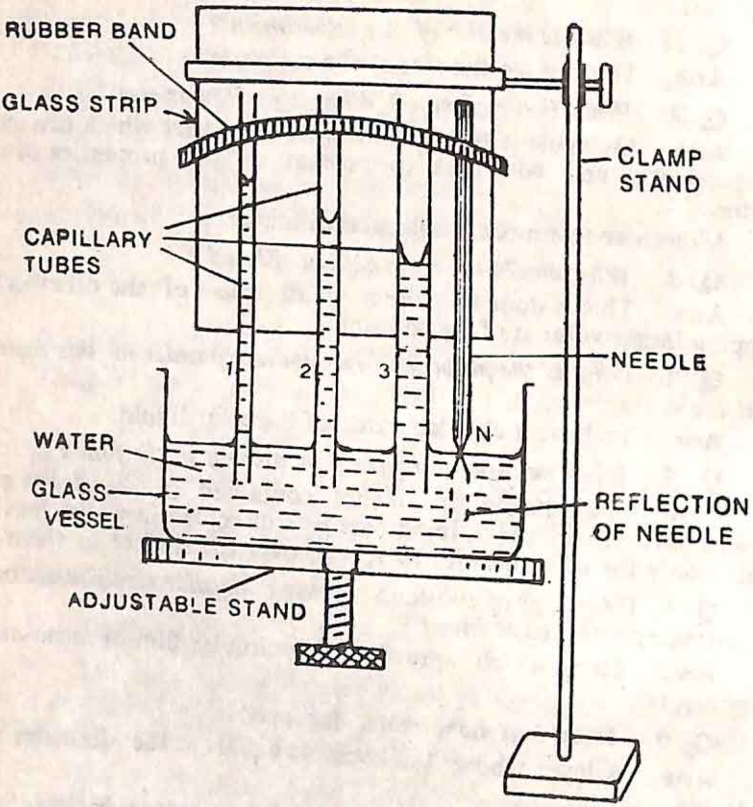


Fig. 28·1. Surface tension of water.

surface is *concave upwards* and the pressure in the liquid just below the meniscus is less than the atmospheric pressure above it by an amount $\frac{2T}{r}$ where T is the surface Tension of the liquid and 'r' is the radius of the tube, which is the same as the radius of curvature of the meniscus Hence the liquid rises in the tube and the weight of the liquid in it balances the difference of pressure.

Let the height of the liquid in the tube from the horizontal surface in the vessel to the tangent plane CD at the bottom of the meniscus be 'h' and 'ρ' the density of the liquid. (Fig. 28 2)

$$\therefore \text{ Weight of the liquid column} = \pi r^2 \left[h + \frac{r}{3} \right] \rho g$$

The surface tension acts along the tangent plane to the liquid surface inwards and its reaction acts outward along AE or BF all along the meniscus which touches the inner surface of the

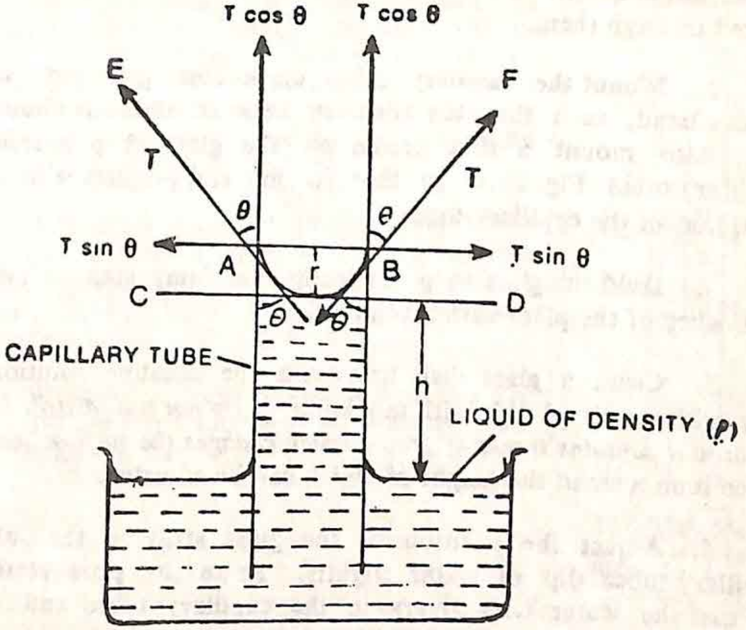


Fig. 28·2.

tube all round the circumference $2\pi r$. The vertically upward components of this force support the weight of the liquid column.

$$\therefore \qquad 2\pi r\, T \cos\theta = \pi r^2 \left[h + \frac{r}{3} \right] \rho\, g$$

or

$$T = r \frac{\left[h + \frac{r}{3} \right] \rho\, g}{2 \cos\theta}$$

For liquids which *wet** *glass*, the angle of contact $\theta = 0$

$$\therefore \qquad \boxed{T = r \frac{\left[h + \frac{r}{3} \right] \rho\, g}{2}}$$

Procedure :

1. Select three capillary tubes of *different* diameters (ranging from 0·5 to 1·5 mm) and cut off their sealed ends with a sharp file or a blade.

2. Clean the capillary tubes carefully first with caustic soda and then with nitric acid, washing out the nitric acid with

---

*This will be so when the bore of the capillary tube is *very fine*. Thus this assumption is only partially true in actual practice.

considerable quantities of water. Then dry the tubes with dry air forced through them.

3. Mount the capillary tubes on a clean glass strip with a rubber band, such that the distance between them is about one cm. Also mount a thin needle on the glass strip parallel to capillary tubes (Fig. 28.1) so that its fine end projects a little less than that of the capillary tubes.

4. Hold the glass strip vertically in a clamp stand. Test the verticality of the plate with a plumb line.

5. Clean a glass dish first with the alkaline solution and then with water. Fill it with tap water. (*Do not use distilled water because it contains traces of grease which changes the surface tension*). Place it on a stand the height of which can be adjusted.

6. Adjust the position of the glass strip so that all the capillary tubes dip in water slightly. Raise the glass vessel and see that the water rises *freely* in the capillary tubes and falls as soon as the glass vessel is lowered.

If water does not fall back readily, it means that the capillary tubes or water or both are not free from contamination. In that case they should be replaced by cleaner ones.)

7. Make the capillary tubes and the needle vertical. Raise the stand holding glass strip so that the needle *just* touches the surface of water in the glass dish. (*Do not touch the surface o water with fingers*).

8. To set the travelling microscope ;

(*i*) Adjust the position of the eye piece so that the cross-wires are clearly visible.

(*ii*) Place the microscope with its horizontal traverse parallel to the plane of the glass plate.

(*iii*) Adjust one of the levelling screws at the base of the microscope so that its scale is vertical and the microscope tube is horizontal. Test with a spirit level.

9. Direct the microscope on the capillary tube having the maximum height of water in it. Focus it by removing parallax between the cross-wires and the image of water column in the capillary tube. Lower the microscope vertically and make sure that it can reach the water level. Rotate the eye-piece of the microscope to make one of the cross-wires horizontal.

Adjust the position of the microscope so that the horizontal cross-wire is approximately at the position M (Fig. 28.3) of the meniscus of water in capillary tube No. 1 having the greatest rise of water.

Work the screw at the top of the microscope till the cross wire is exactly in the position M Read the vertical scale.

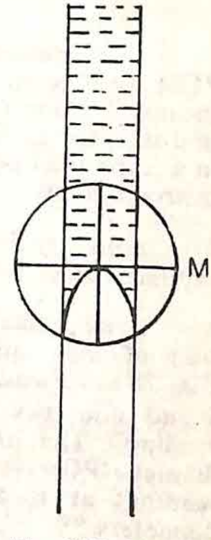


Fig. 28·3. Magnified view of capillary tube No. 1.

10. Move the microscope along the horizontal scale and bring it in front of tube No. 2. Adjust its height, focus it on the lower meniscus of water and note the reading. Similarly take the reading by focusing the microscope on the lower meniscus of water in the capillary tube No. 3.

11. Now bring the microscope in front of the needle and lower it till the cross-wire is in the position N as shown in Fig. 28.4 symmetrically between the tip of the needle and its image in water. Read the vertical scale again

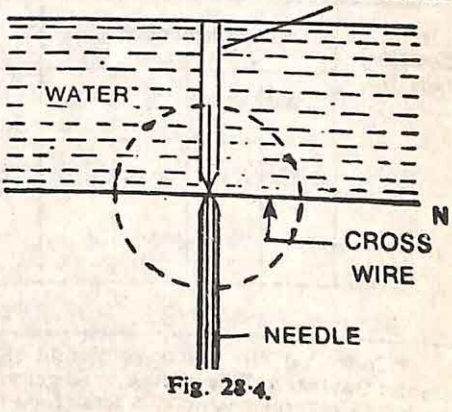


Fig. 28·4.

12. Mark with pen the position of the lower meniscus of water in each of the tubes.

13. *Measurement of the diameter* : *Cut the tube No. 1, at the ink mark by means of a sharp glass cutting file and find its internal diameter by clamping it in a horizontal position and focusing the microscope over it.

Similarly find the diameters of the capillary tubes No. 2 and 3.

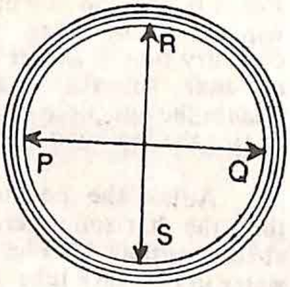


Fig. 28·5.

A magnified view of the cross-section of the capillary tube is shown in Fig. 28.5. Focus the microscope first at P and note the reading. Now focus it at Q and again note the reading. The difference between these two readings gives the diameter PQ. Rotate the tube through a right angle and take the readings at R and S. Find the mean of the values of the two diameters.**

14. Find the temperature of water used.

Observations and Calculations :

Temperature of water $=t=$........C°

Density of water at t°C =........g/cm^3

Angle of contact for water$=\theta=$........

Diameter of the Tube :

Vernier constant of the microscope=........cm

Capillary tube No.	*Microscope reading at*				*PQ*	*RS*	*Mean diameter*	*Mean radius (r)*
	P	*Q*	*R*	*S*				
1.	...cm	...cm	...cm	...cm	...cm	...cm	...cm	...cm
2.								
3.								

---

*Under no circumstances should the tube be broken by bending it otherwise the fracture will be uneven, To get a clear fracture perpendicular to the length of the capillary, make a scratch on the tube at the ink mark with a tiny file and apply the tension *along* the length of the tube.

**The four readings should vary (if at all) amongst themselves by a very small amount.

Height of the liquid column and surface tension :

Capillary tube No.	*Reading at*		*Height (h)*	*Mean radius (r)*	*Surface tension* $T=r\frac{\left[h+\frac{r}{3}\right]\rho g}{2\cos\theta}$
	M	*N*			
1.	……cm	……cm	……cm	……cm	……dynes/cm
2.					
3.					

Mean value of Surface Tension =………dynes/cm

Value from tables at room temperature=………dynes/cm

% error=……,.

Result :

The surface Tension of water at…………………C°

=………dynes/cm

Precaution :

1. The tubes should be of *uniform* bore throughout.

2. The tubes must be vertical and sufficiently apart.

3. Water should rise freely in the tubes.

4. Do not use wax for fixing the tubes on the glass plate. Instead use a thin rubber band.

5. The surface of water should not be touched with hand and the vessel containing it should not be greasy.

6. The diameters of the tubes should be measured accurately at the points where the liquid level stands.

7. The diameter should be measured accurately in two mutually perpendicular directions.

8. The tip of the needle should just touch the water surface and not dip into it.

9. The back-lash error with the microscope should be avoided by always turning the screw in the same direction.

Sources of Error :

(*i*) Probable contamination of the liquid surface as also the contamination of the capillary tube ; the cleaning of which is tedious.

(*ii*) Chief source of error is the determination of the radius of the bore as it is extremely difficult in practice to have a capillary tube of absolutely uniform bore throughout.

Exercise

Compare the radii of the two capillary tubes by noting the rise of the liquid in them.

Hints : If the liquid wets glass

$$T=\frac{r\left[h+\frac{r}{3}\right]\rho g}{2}$$

If the radius (r) is small then $\frac{r}{3}\ll h$

$$\therefore \qquad T=\frac{r h \rho g}{2}.$$

If h and h' are the heights of liquid columns in the two capillary tubes of radii r and r' respectively, then

$$2T=rh\,\rho g=r'\,h'\,\rho g$$

or
$$\boxed{\frac{r}{r'}=\frac{h'}{h}}$$

ORAL QUESTIONS

Q. 1. *What do you mean by surface tension ?*

Ans. The free surface of every liquid behaves as if it were in a state of tension having a natural tendency to contract. This tension or pull in the surface of a liquid is called *Surface Tension.* If a line be imagined to be drawn on a liquid surface, a force acts on each side of this line, the direction of the force being tangential to the surface and perpendicular to the line. *The magnitude of this force acting per unit length of this line is known Surface Tension.*

Q. 2. *Is there any other way of defining surface tension ?*

Ans. Yes. The surface tension may also be defined as numerically equal to the work done in enlarging the surface of a liquid by unit amount under isothermal conditions.

Q. 3. *Why should work be at all done in enlarging the surface of a liquid ?*

Ans. As the surface is enlarged, the surface density of the molecules, *i.e.*, the population of the molecules per cm^2 of the free surface, is diminished. Consequently more molecules must be brought to the surface from the interior of the liquid in order to keep this population constant. To give room to these newcomers on the surface, the adjacent molecules have to be separated. Thus work is necessary to separate the molecules against the cohesive forces which are existent between the molecules.

Q. 4. *What happens to the energy which constitutes the work done ? Is it wasted ?*

Ans. No. This energy is not wasted, but is stored in the surface molecules in the form of enhanced potential energy. It is due to this fact that the surface of a liquid behaves like a stretched membrane.

Q. 5. *Why mercury does not stick to the finger, while water does under the same conditions ?*

Ans. In the case of mercury the force of cohesion between mercury and mercury is *greater* than the force of adhesion between mercury and finger. Hence the molecules of mercury just in the neighbourhood of the finger are unable to be torn off from the parent mass of the liquid. Consequently mercury does not stick to the finger.

In the case of water, however, *the adhesive forces are stronger than the cohesive forces* ; hence when the finger is dipped in water, the molecules of water close to the finger leave the liquid and stick to the finger. Consequently the finger gets wetted.

Q. 6. *What is the nature of these cohesive forces ?*

Ans. These forces are not of the same nature as gravitational forces but arise on account of the interaction of the electrical fields which surround atoms. *These are very short-range forces.* Their influence is experienced in a very narrow range of the order of 10^{-5} mm. Thus, if two molecules be separated by a distance greater than this, the cohesive force existent between them becomes insignificantly small.

Q. 7. *Why water rises in a capillary tube ?*

Ans. In the capillary tube (Fig. 28·6), the water meniscus is concave. Now, as the surface of a liquid behaves like a stretched membrance and wants to contract as much as possible, the surface in the tube becomes straight as shown by the dotted line. Hence space below this dotted line is filled by water.

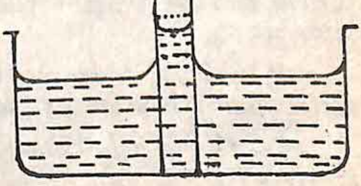


Fig. 28·6. Why water rises.

Now, the meniscus cannot remain straight, it again becomes concave. Due to surface tension it again becomes straight. This process of straightening and curving of the meniscus continues and consequently water rises in the tube.

Q. 8. *Should the water level continue to rise indefinitely ?*

Ans. No. Each time when the surface straightens and water rises, the surface has to pull a column of water. Now, when the weight of the water column becomes so great that the tendency of the surface to straighten up is checked, the water level ceases to rise. Hence the rise of water column is not indefinite.

Q. 9. *Why does water not rise in an ordinary glass tube—, why in a capillary tube ?*

Ans. The height of water column in a glass tube depends upon the weight of the liquid contained in this column. This obviously depends upon the radius of the tube. In a capillary tube the radius is exceedingly small, hence the weight of the liquid for balancing surface tension is attained in a good length of the tube. In a wide tube or ordinary glass tube, the same weight of water will be contained in an extremely small length of the tube and the rise may not even be perceptible.

Q. 10. *What is the relation between Surface Tension and Weight of the Liquid Column supported by it ?*

Ans. $2\pi r \, T \cos\theta = \pi r^2 g \left[h + \frac{r}{3} \right] \rho$

[See Theory of the Expt. above]

Q. 11. *Define angle of contact.*

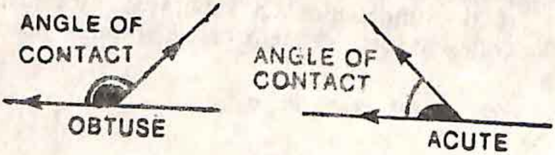


Fig. 28·7.

Ans. The angle of contact (Fig. 28.7) between a liquid and a solid is the angle measured within the liquid, between the solid surface and the tangent plane to the liquid surface at the point of intersection.

It is *obtuse* in the case of liquids which do not wet the solid surface and it is *acute* for those liquids which wet the solid surface.

Q. 12. *What is the angle of contact for pure water-glass interface ?*

Ans. Practically *zero.*

Q. 13. *Why does water assume concave meniscus in a capillary tube ?*

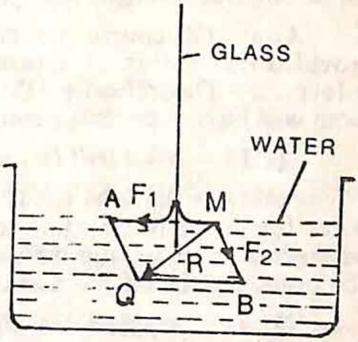


Fig. 28·8. Concave meniscus.

Ans. In Fig. 28.8 consider a molecule 'M' of water. Let force of adhesion (F_1) between the water molecule M and glass molecules and the force of cohesion (F_2) between the water molecule (M) and the rest of the water molecules be represented in magnitude and direction by $\overrightarrow{MA}$ and $\overrightarrow{MB}$ respectively. In this case of water-glass interface, $F_1 > F_2$ consequently the resultant R represented by the diagonal $\overrightarrow{MQ}$ takes the direction as shown. This is a force experienced by the surface just in the vicinity of this molecule M. Now, for equilibrium, the surface of the liquid must be at right angles to the resultant force acting on it. This can be achieved by the *surface curving upwards* as shown. Now if we consider the other wall of the capillary tube, it is clear that inside it the surface must curve upwards for both the walls. Hence the surface must be concave.

Q. 14. *Do all liquids show this behaviour, i.e., assume concave meniscus in a capillary tube ?*

Ans. No. The case discussed in (*Q. 13*) is true only for those liquids and those surfaces, in contact with them, for which *force of adhesion is greater than the force of cohesion.* For those liquids, for which reverse is the case, the surface is *convex.* For example mercury and glass surface.

Q. 15. *If a capillary tube is dipped in mercury, what will you observe ?*

Ans. The level of mercury in the capillary tube will go on sinking.

Q. 16. *When will it stop sinking ?*

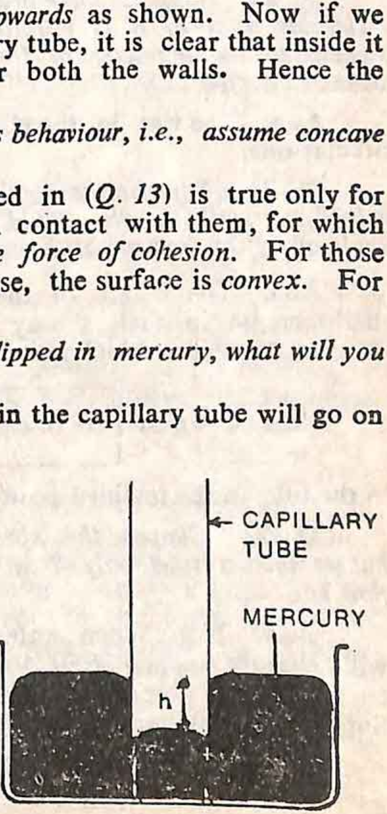


Fig. 28·9.

Ans. When the tendency of the meniscus to contract downwards is counterbalanced by the hydrostatic pressure exerted by the mercury column due to difference '*h*' (Fig. 28.9) between the levels of mercury in the outside vessel and inside the capillary tube.

Q. 17. *Can we use a tube in this experiment whose bore may not be uniform throughout ?*

Ans. Of course we can use a capillary tube of this type provided the radius is measured at the place where the meniscus is formed. Theoretically there is no difficulty but practically, there will be one probable source of error.

Q. 18. *What will be that probable source of error ?*

Ans. If the tube is cut even slightly up or down the desired point for the determination of the radius of the tube, there will be serious error in the value of 'r', the radius of the tube. Hence the result will be off the mark.

Q. 19. *Suppose we dip a number of tubes of the same bore but made of different materials, in the same liquid, will we get the same rise in all ?*

Ans. No ; because the interfacial surface tension will be different in each case.

Q. 20. *Suppose there are two tubes A and B. The bore of A is circular, that of B is elliptical—the major axis of the ellipse is equal to the diameter of the former. How will the levels in the tubes be affected ?*

Ans. The rise in the elliptical tube will be *more* than in the circular one.

Q. 21. *Suppose the liquid occupies 5 cm of the tube when it is held vertically in the liquid. Now, if the tube is inclined at an angle of 60° to the vertical, how will the rise in tube be affected ?*

Ans. The length of the liquid column occupying the tube shall increase in such a way that the vertical height is even now 'h'. So the angle of inclination is 60°, we have

$l \cos 60° = h$ or $\boxed{l = 10 \text{ cm}}$ where l is the length of the liquid in the tube in the inclined position.

Q. 22. *Suppose the expected rise in a capillary tube is 10 cm ; but we have a tube only 8 cm long. When dipped, will water over flow ?*

Ans. No. When water shall reach the top of the tube, it will slightly spread itself there thereby adjusting its radius of curvature to a new value so that equilibrium is again established. Suppose the new radius of curvature is 'r_1', then

$$r_1 \times 8 = r \times 10$$

Thus with a tube of sufficient length r_1 will be greater than r or the meniscus shall become less concave.

Q. 23. *What is the unit of surface tension? What are the dimensions of surface tension?*

Ans. Unit—newtons per metre (N/M)

dimensions—M^1T^{-2}.

Q. 24. *How does surface tension vary with temperature?*

Ans. The surface tension of all liquids decreases as the temperature rises.

Q. 25. *What is the effect of contaminations on the surface tension of a liquid?*

Ans. All sorts of impurities, contaminations and dissolved substances *lower* the surface tension of a liquid.

Q 26. *Why rain drops are spherical in shape and bigger drops are flat?*

Ans. Because in the case of rain drops the surface tension is greater than the force of gravity and in the case of bigger drops, the weight of the drop is greater than the upward force of surface tension.

Q. 27. *What are the factors that affect surface tension of a liquid?*

Ans. (*i*) Temperature, (*ii*) Contaminations ; impurities and dissolved substances.

Experiment 29.

To study the relationship between pressure and temperature of a sample of air at constant volume and hence determine the coefficient of increase of pressure of air at constant volume by Jolly's constant volume air thermometer.

Apparatus :

Jolly's constant volume air thermometer, a plumb line, a beaker, a thermometer, ice, a funnel, a set square, and a heating arrangement.

Theory :

A simple form of constant volume air thermometer (Fig. 29.1) due to Jolly consists of a glass bulb C at the end of a capillary tube about 1 mm in diameter. The tube is bent twice at right angles and joined by a thick-walled rubber tubing to an open glass tube D. The part of the capillary tube attached to the bulb beyond the second bend is fixed vertically on a vertical wooden board fixed on a horizontal iron base provided with three levelling screws. A millimetre scale is fixed on the vertical board with its zero mark in the lower most position. The open tube which is also in front of the scale can slide up or down along the vertical board and can be clamped at any position. The rubber tube and the open tube

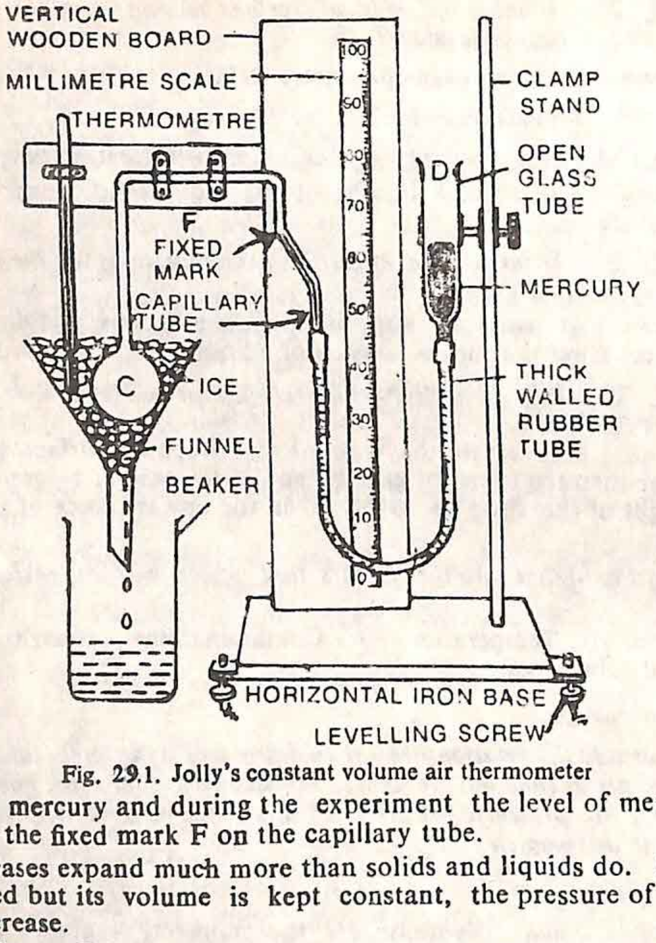


Fig. 29.1. Jolly's constant volume air thermometer

contain mercury and during the experiment the level of mercury is kept at the fixed mark F on the capillary tube.

Gases expand much more than solids and liquids do. If a gas is heated but its volume is kept constant, the pressure of the gas will increase.

The coefficient of increase of pressure at constant volume is defined as the increase in pressure per unit pressure at 0°C, per degree centigrade rise of temperature, volume remaining constant.

If P_0 is the pressure at 0°C and P_t the pressure at t°C, then the coefficient of increase of pressure at constant volume γ_v is given by

$$\gamma_v = \frac{P_t - P_0}{P_0 \times t}$$

or

$$P_t = P_0 (1 + \gamma_v t)$$

The value of coefficient of increase of pressure at constant volume for all gases is $\frac{1}{273}$ or 0·00367.

In the case of gases the lower temperature must be 0°C and not the room temperature as an appreciable error will be caused due to the very large value of the coefficient of expansion of a gas *i.e.*, $\frac{1}{273}$ or 0·00367, which is approximately 10 times the coefficient of cubical expansion of water and 100 times the coefficient of cubical expansion of copper.

It is possible to determine the coefficient of expansion of a gas from readings of pressure at any two temperatures. If P_0, P_1 and P_2 are the pressures at temperatures 0°C, t_1°C and t_2°C respectively and γ_v is the coefficient of increase of pressure at constant volume, then

$$P_1 = P_0 (1 + \gamma_v t_1) \qquad \ldots(i)$$

$$P_2 = P_0 (1 + \gamma_v t_2) \qquad \ldots(ii)$$

Dividing (*ii*) by (*i*), we have

$$\frac{P_2}{P_1} = \frac{1 + \gamma_v t_2}{1 + \gamma_v t_1}$$

or

$$\gamma_v = \frac{P_2 - P_1}{P_1 t_2 - P_2 t_1} \qquad \ldots(iii)$$

Procedure :

1. **Set the apparatus vertical as follows** :

Suspend a plumb line in front of the scale. Adjust the levelling screws at the base so that the plumb line is parallel to the edge of the scale and equidistant from it at all points. *Adjust the central screw to make it parallel to the edge and the two side screws to make it equidistant.*

2. Make a fixed mark, F below the bend on the capillary tube by pasting a piece of gumpaper on it. *The mark is made below the bend so that mercury may not enter the bulb C due to negligence.*

3. Note the atmospheric pressure from Fortin's barometer as follows :

(*i*) Find the vernier constant of the Fortin's barometer.

(*ii*) Adjust the screw at the base of the barometer so that the mercury surface just touches the ivory tip.

(*iii*) Place the eye at the same level as the meniscus of mercury to avoid parallax and note the reading.

4. Bring the open tube D to the lowest position. Place a glass funnel below the bulb C. Wash ice in running water and pound it in a *clean duster*. Surround the bulb *completely* with pounded ice and suspend from a clamp a thermometer with its bulb dipping in ice near the glass bulb. Wait for about 10 minutes so that the air in the bulb attains the temperature of ice. Raise the tube D till the mercury level in the capillary stands at the fixed mark F.

5. Wait for few minutes. If the air in the bulb has attained the temperature of ice and there is no leakage in the apparatus, then the mercury level will remain stationary at F.

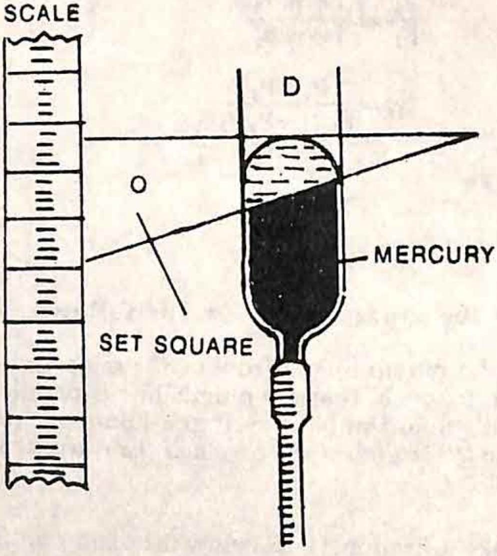


Fig. 29.2

In order to note the reading of the upper meniscus of mercury in the open tube D, place a set square such that one of its perpendicular sides is horizontal and the other presses against the side of the metrescale as shown in Fig. 29.2. Move it till the horizontal edge is at the level of mercury. Place the eye at the same height *inorder to avoid error due to parallax* and note the scale reading against the horizontal edge of the set square. Similarly, note the reading of the mercury level on the scale against the fixed mark F.

6. Remove ice and the funnel surrounding the bulb C. Place the bulb in the steam bath as shown in Fig. 29.3. When the bulb remains surrounded with steam for about 10 minutes, the air in the bulb will attain the temperature of steam. See that the thermometer records a constant temperature. Raise the tube D so that mercury in the capillary tube again stands at the fixed mark F. Wait for a few minutes more. If the air in the bulb has attained the temperature of steam, the mercury level will remain stationary at F. Note the scale reading against the level of mercury in the tube D, with the help of a set square as explained in step 5 of the procedure. *Bring the tube D to its lowest position and then remove the burner.*

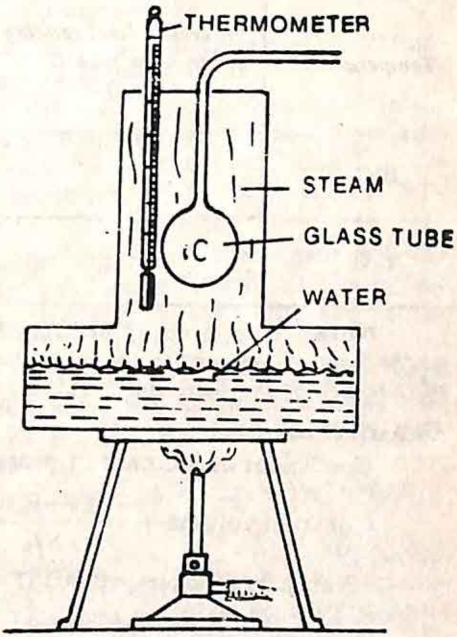


Fig. 29.3

7. Note the atmospheric pressure again and find the temperature of steam at this pressure.

Observations

Atmospheric pressure in the beginning = ...cm of Hg

Atmospheric pressure at the end = ...cm of Hg

Mean atmospheric pressure P = ...cm of Hg

= ......mm of Hg

Temperature of steam $= t_2 = 100 - {}^{\cdot}0367\,(760 - P)$

$= \ldots\ldots{}^{\circ}\mathrm{C}$

where P is the atmospheric pressure in *millimetres of mercury.*

Reading of the mercury level at

the fixed mark F = x = ...cm

Temperature	Mercury level reading in open tube D (y)	Head of pressure $\pm(y-x)=\pm h$	$P\pm h$
0°C			$P_0=$
t_2°C			$P_2=$

Note. *The head of pressure h is positive or negative according as the level of mercury in the open tube D is above or below the level of mercury at the fixed mark F.*

Calculations

Coefficient of increase of pressure at

$$\text{Constant volume } \gamma_v=\frac{P_2-P_0}{P_0\times t_2}=\ldots\ldots\ldots$$

Value from tables=0·00367

% error=........

Precautions

1. The enclosed air should be completely dry.

2. The pressure in the beginning and at the end of the experiment should be the same.

3. The mercury should not be sticking to the sides of the glass tube.

4. The reading of mercury level should be taken when the temperature remains constant for several minutes and the mercury remains at the fixed mark F.

5. The open tube should be considerably lowered after the reading at the steam temperature has been taken otherwise mercury will rush into the bulb when the burner is removed.

6. The bulb should be surrounded by steam and should not be immersed in water.

7. The bulb should be *completely* surrounded by *pounded* ice.

Sources of Error

1. Glass bulb expands and the volume of air increases, the *volume, therefore, does not remain constant.*

2. The air in the capillary tube is not at the same temperature as the air in the bulb.

3. The enclosed air may not be completely dry.

Exercises

1. To plot a graph between temperature and pressure at constant volume using Jolly's apparatus.

Hints : (*i*) Surround the bulb C in ice and note the observations as explained in step 4, **Experiment 29**.

(*ii*) Remove ice and surround the bulb with water in the brass mug at room temperature. Raise the position of D so that mercury stands at the fixed mark F in the capillary tube. Note the reading of the mercury level in the tube D at the room temperature.

(*iii*) Heat water to the boiling point and again note the reading of mercury in the open tube D after adjusting its position so that mercury again stands at F in the capillary tube.

(*iv*) Remove the burner and allow the water to cool. Note the reading of mercury level in the tube D at temperatures 90°, 80°, 70° etc., by keeping the temperature constant at these points.

(*v*) Plot a graph between the pressure P and the temperature t.

(*vi*) From the graph calculate the values of P_1 and P_2 corresponding to any two temperatures t_1 and t_2 respectively and then calculate the coefficient of increase of pressure from the relation :

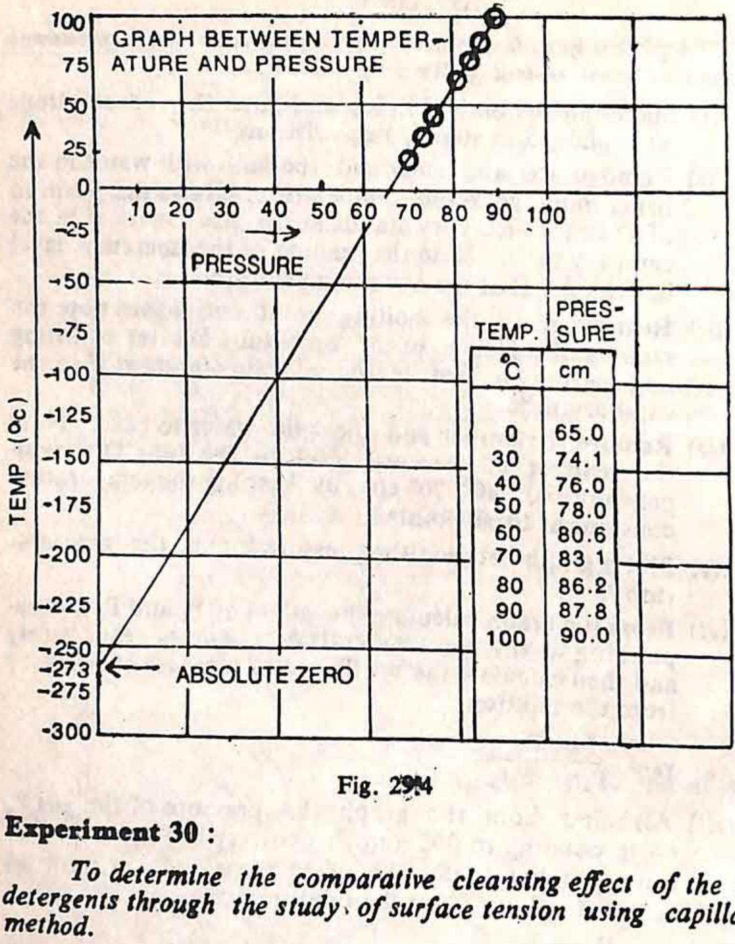
$$\gamma_v = \frac{P_2 - P_1}{P_1 t_2 - P_2 t_1}.$$

(*vii*) Also find from the graph the pressure of the gas P_0 corresponding to 0°C and P_{100} corresponding to 100°C. Calculate the coefficient of increase of pressure at constant volume from the relation

$$\gamma_v = \frac{P_{100} - P_0}{P_0 \times 100}.$$

2. Find the value of absolute zero by taking observations with Jolly's constant volume air thermometer.

Hint : Plot a graph between the pressure P and the temperature t representing pressure along X-axis and temperature along Y-axis taking zero value at the origin in both the cases. The position of X-axis is selected in such a manner that it is possible to go upto 100°C on the positive side and upto 300°C on the negative side. The graph is a straight line as shown in Fig. 29.4, Produce the curve to meet the Y-axis. The point at which it meets the Y-axis gives the value of the absolute zero. This is because corresponding to this temperature the pressure of the enclosed gas is zero.



TEMP. °C	PRESSURE cm
0	65.0
30	74.1
40	76.0
50	78.0
60	80.6
70	83 1
80	86.2
90	87.8
100	90.0

Fig. 294

Experiment 30 :

To determine the comparative cleansing effect of the different detergents through the study of surface tension using capillary tube method.

Apparatus :

A capillary tube, beakers, spring balance, a glass strip, a thin rubber tube band, a needle, a clamp stand, a travelling microscope, a thermometer, an adjustable stand, different detergents and distilled water.

Theory :

Same as in Expt. 28.

As the surface tension of water is reduced by dissolving a detergent, the greater the cleaning strength of a detergent, the less is its capillary rise.

Procedure :

1. Take equal weights of different detergents with the help of a spring balance. Dissolve them in *equal volume* of distilled water, kept in different beakers.

2. Compare the capillary rise of different solutions by dipping a capillary tube in each solution turn by turn *as per procedure explained in* **Experiment 28**.

The capillary tube must be thoroughly cleaned and rinsed by distilled water. It must also be rinsed by distilled water before dipping it in each solution.

As the surface tension of water is reduced by dissolving a detergent, the greater the cleaning strength of a detergent, the less is its capillary rise

Note :

An *alternative method* is to have several capillary tubes of same internal diameter. These are cut from the same capillary tube. Then one capillary tube is dipped in each beaker. The beakers should be of same size and should contain equal volumes of different detergent solutions, so that the different capillary rise of different detergents is clearly visible even from a distance.

Observations and Calculations :

Height of liquid column in the capillary tube :

Detergent Solution	*Readings of microscope at*		*Height (or capillary rise) (h) in cm*
	M	*N*	
(i)			
(ii)			
(iii)			
(iv)			

Result :

(*a*) The cleansing effect of detergent solution........... is the maximum.

(*b*) The cleansing effect of detergent solution...........is...........

(*c*) The cleansing effect of detergent solution...........is...........

(*d*) The cleansing effect of detergent solution........... is the least of all.

Precautions :

Same as in Expt. **28**.

Sources of Error :

Same as in Expt. 28.

ORAL QUESTIONS

Same as in Expt. 28.

[Same as in experiment 32.]

Experiment 31.

To determine the surface tension of a liquid (water) by the drop-weight method.

Apparatus :

Funnel, a short glass tube of internal diameter 4 mm or 5 mm, a piece of rubber tubing which can fit tightly on the glass tube, paraffin wax, small beaker or crucible, screw type pinch cock, travelling microscope, the given liquid (say water).

Theory :

Let us make a simplifying assumption that the drop falling from a circular orifice has cylindrical form when it is about to break away from the tube. If r is the radius of the orifice, the excess pressure inside the drop at this section is $\frac{T}{r}$ due to cylindrical curvature where T is the surface tension of the liquid. This produces a downward thrust on the drop of $\frac{T}{r}.\pi r^2 = T\pi r$. If m is the mass of the drop, the total downward forces are $T\pi r + mg$.

These forces are balanced by the upward forces of surface tension round the circle of contact. This force is $2\pi r$ ‾

Hence,

$$2\pi r.T = T\pi r + mg$$

or

$$T = \frac{mg}{\pi r}$$

The above expression is deduced on the assumption that (*i*) the drops break away under ideal statistical conditions, and (*ii*) shape of liquid at the orifice is cylindrical when the drop is about to break away. The problem, however, is complicated by dynamical considerations and complex shape of drops. Taking these into account Rayleigh has shown that a closer approximation is given by the formula

$$\boxed{T = \frac{mg}{3{\cdot}8\, r}}$$

Procedure :

1. Place the funnel in a ring as shown in Fig. 31.1. Fix the glass tube to the lower end of the funnel with a rubber tubing. Attach the pinch cock on the rubber tube, to regulate the liquid flow. Make a thin coating of wax on the dropping end of the glass

tube. This ensures that the drops fall out from its internal surface only. For this purpose it is also necessary that the glass tube is cut flat. If the flat end is not obtained by cutting only, it may be grounded flat using a very fine emery powder or sand. In no case should it be fire polished.

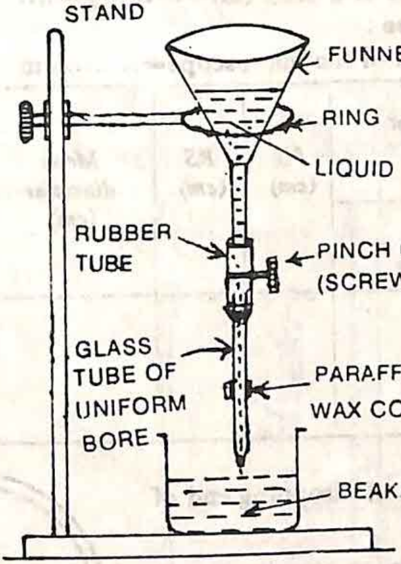


Fig. 31.1

2. Place the liquid whose surface tension is to be measured in the funnel. Regulate the flow of the liquid so that the drops break away at the rate of about one per minute. Collect the counted number of drops in a beaker of known mass and weigh the beaker with the collected liquid. From this, the average mass of the drop is found out.

3. The mean radius, ***r***, of the end of the glass tube is determined by means of a travelling microscope as explained in Expt. 28.

4. Calculate the surface tension, T, using the formula

$$T = \frac{mg}{3{\cdot}8\, r}$$

where *m* is the average mass of a drop and *g* is the acceleration due to gravity.

Note : For more accurate measurement, the rate of breaking away of the drops should be even slower than what has been suggested above. It is better to keep it one drop every four minutes but it may take the experiment too much time consuming. When time is important, students may do a tolerable experiment even by collecting 50 drops in about 10 minutes.

Observations and Calculations :

Mass of beaker=.........g

Mass of beaker and collected liquid=.........g

Number of drops=.........

Average mass of a drop (m)=.........g=.........kg

Diameter of the tube :

Vernier constant of the microscope=.........cm

Microscope reading at (in cm)				*PQ (cm)*	*RS (cm)*	*Mean diameter (cm)*	*Mean radius (cm)*
P	*Q*	*R*	*S*				
							.........(cm)
							.........(cm)

∴ Mean radius of dropping end of glass tube$=r=$......m

Acceleration due to gravity at...... (from tables)$=g=$......ms^{-2}

∴ Surface tension of the liquid

$$=T=\frac{mg}{3{\cdot}8\,r}=\ldots\ldots \text{N}/m$$

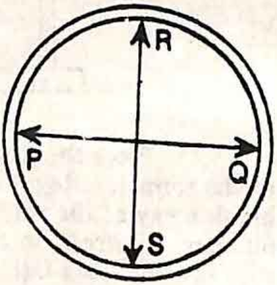


Fig. 31.2

Result :

The surface tension of the liquid=......Nm^{-1}

Precautions :

1. The glass tube should be cut flat.
2. The drops of the liquid should fall out from the internal surface of the glass tube only.
3. The flow of liquid should be regulated in such a way that the drops break away at the rate of about one per 4 minutes or one per minute if time is important.
4. The glass tube should be of *uniform* bore throughout.
5. The diameter of the flat end of the glass should be measured accurately in two mutually perpendicular directions.

ORAL QUESTIONS

(Same as in Experiment 28.

SECTION G

Experiment 32 :

To determine the spring constant of a given spring.

Apparatus :

Light spiral spring, metre scale, two clamps and stand pointer for spring, hanger with slotted weights.

Theory :

The shape of a body changes when an external force is applied. Molecular forces tend to oppose this change. As soon as external force is removed, the body regains its original shape, if the applied force is within the elastic limit.

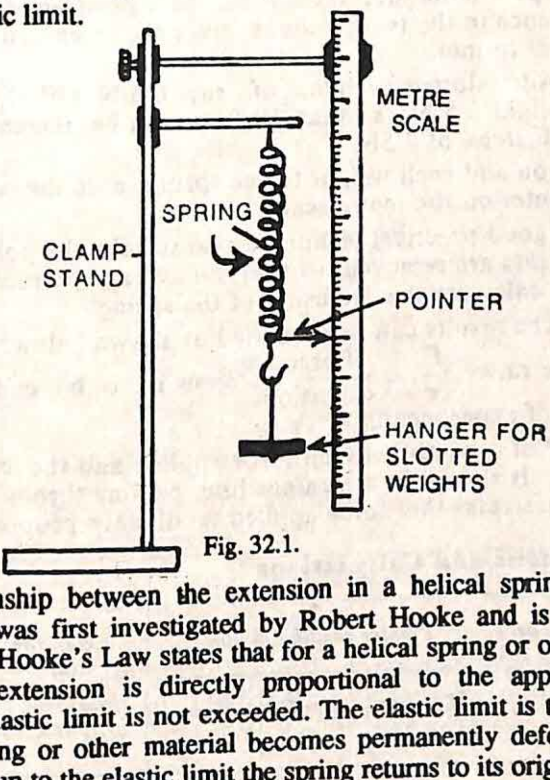


Fig. 32.1.

The relationship between the extension in a helical spring and the applied force was first investigated by Robert Hooke and is known as Hooke's Law. Hooke's Law states that for a helical spring or other elastic material, the extension is directly proportional to the applied force, provided the elastic limit is not exceeded. The elastic limit is the point at which the spring or other material becomes permanently deformed. For forces applied up to the elastic limit the spring returns to its original length or shape when the force is removed.

$\therefore F \propto e$ where F = applied force

and e = extension

$F = \text{constant } e$

or $F = Ke$ where K = spring constant

$$\therefore \text{ Spring constant} = K = \frac{F}{e} = \frac{\text{Force}}{\text{extention}} = \frac{mg}{e}$$

where m = mass

g = acceleration due to gravity.

Its SI unit is Nm^{-1}

Procedure :

1. Set up the apparatus as shown in Fig. 32.1

2. Fix a pin to the lower end of the spring to act as a pointer and record the position of the pointer on the metre scale when no force is applied.

This arbitrary zero position should be checked at the end of the experiment to ensure that the maximum force applied to the spring did not exceed the elastic limit.

3. Attach a weight hanger (which applies a known force $F=0{\cdot}5$ N to the spring) to the lower end of the spring. When the spring stops oscillating, record the new position of the pointer. The difference in the two readings gives the extension *e* which can be recorded in mm.

4. Add slotted weights of, say 0·5 N (50 g) to the hanger (also of weight 0·5 N) so that the force can be increased from 0 to 3N *in equal steps* of 0·5N.

As you add each weight to the spring, note the reading of the steady pointer on the metre scale.

It is good practical technique also to take the pointer readings as the weights are removed so that an average pointer reading can be used to calculate the extension of the spring.

5. The results can be tabulated as shown below. A calculation of the ratio $\frac{F}{e}=\frac{\text{Force}}{\text{extension}}$ show it to be constant within the limits of experimental error.

6. Plot a graph between force applied and the corresponding extension. It should be a straight line passing through the origin. This demonstrates that force applied is directly proportional to the

Observations and Calculations :

S. No.	*Force (F) in N*	*Pointer reading in mm*			*Extension (e) in mm for......N*	*Spring constant* $K=\frac{F}{e}$ *(in* Nmm^{-1}*)*
		Loading	*Unloading*	*Mean*		
1	0			A		
2	0·5			B		
3	1·0			C	(D−A)=...mm	
4	1·5			D	(E−B)=...mm	
5	2·0			E		
6	2·5			H	(H−C)=...mm	
7	3·0			G	(G−D)=...mm	
				Mean Value of e=	.........mm	

extension. From the graph, calculate K. This is $\frac{b}{a}$ (See Fig. 32.2).

From the graph (Fig. 31.2) ;

The elastic constant of spring $K = \frac{b}{a} =$......N/mm

$=$......N/m

Result :

(1) K=......Nm^{-1} is the force applied on the spring per metre extension.

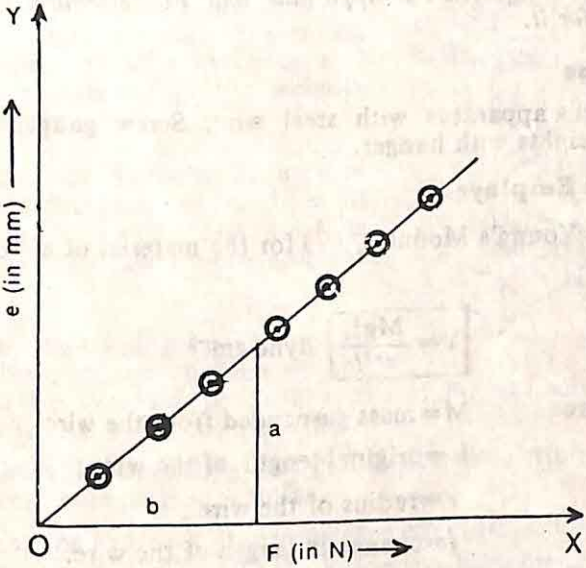


Fig. 32.2

2. Since the graph of F against *e* is a straight line passing through the origin, then the extension of the spring is directly proportional to the tension in the spring (Hooke's law).

Precautions :

1. The spring should be suspended in such a way that its axis is vertical and the scale should be adjusted parallel to its axis.

2. The spring should not be loaded beyond elastic limits.

3. The loading or unloading should be in equal steps and should be done carefully and gently. After each addition or removal of the load, wait for 1 minute in order to allow (*i*) the spring to increase or decrease in length and (*ii*) the oscillations to subside.

4. The pointer should be so adjusted that it is not just touching the scale, otherwise it will rub against the latter and correction reading of extension shall not be obtained.

Sources of Error :

1. The weights used may not be standard weights.
2. Error may occur in reading the pointer positions.

ORAL QUESTIONS

Same as in Experiment 33

Experiment 33

To study graphically the stress—strain relationship for a steel wire using Searl's Apparatus and to determine the Young's Modulus for it.

Apparatus :

Searl's apparatus with steel wire, Screw gauge, Metre-rod, Slotted weights with hanger.

Formula Employed :

The Young's Modulus (Y) for the material of a wire is given by

$$\boxed{Y=\frac{MgL}{\pi r^2 l}} \text{ dyne cm}^{-2}$$

where M=mass suspended from the wire ;

L=original length of the wire ;

r=radius of the wire ;

l=change in length of the wire.

Theory :

If a wire of length 'L' has a load of 'M' gm suspended from its lower end, then a force equal to Mg dynes acts vertically along the axis of the wire and consequently the wire extends in length a little. Let this extension be 'l' cm. Then the longitudinal strain produced in the wire is l/L. If the area of cross-section of the wire be A, then the normal stress set up in the wire in the equilibrium state is given by Mg/A. Then, from definition, the Young's Modulus is given by

$$Y=\frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}}=\frac{Mg/A}{l/L}=\frac{Mg\,.\,L}{A\,.\,l}$$

If the radius of the wire be 'r', then $A=\pi r^2$

and $$Y=\frac{Mg\,.\,L}{\pi r^2\,.\,l}$$

Procedure :

(*i*) First of all determine the pitch and the least count of the micrometer screw. Hang the hanger 'H' in the hook 'B' of the frame F_2 connected to the experimental wire '*b*' to remove any kinks present in it and to keep it[*] taut. Also suspend the dead weight 'W' from the other hook A to keep the reference wire '*a*' straight.

Now without taking any readings add a few weights, one by one, then remove them one by one. This [**]operation should be repeated twice or thrice and care should be taken that the weights are placed gently.

(*ii*) Find the diameter of the wire '*b*' with the screw gauge at six different places. At each point, measure the diameter in two mutually perpendicular directions.

(*iii*) Now by turning the micrometer screw (M) bring the air bubble of the spirit level in the centre and note the reading on the pitch scale and the disc scale.

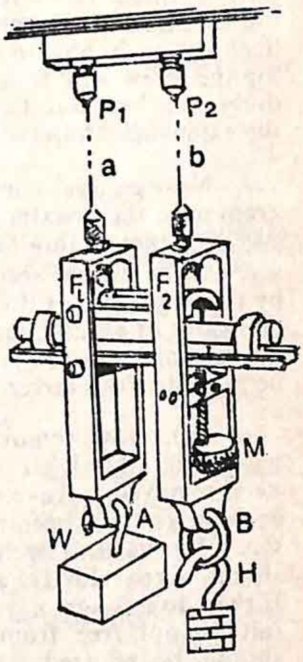


Fig. 33.1 Searle's apparatus

(*iv*) Calculate the *breaking* load and hence permissible load for the given steel wire by the formula.

Breaking load = Breaking stress × area of cross-section in cm^2

∴ [†]Permissible load = $\frac{1}{2}$ of the calculated breaking load. *Do not place the load on the wire more than the permissible load to keep the wire within elastic limits.*

---

[*] If this is not done the values of the first few extensions shall be erroneous.

[**] A wire shows some irregularity in extension when it is loaded for the first time ; but when the loading and unloading is carried on for some time, the wire attains a steady state and the behaviour regarding extension becomes uniform. This indicates that the wire which is strained for the first time must have developed a structural change in itself which could not recover for some time. This has been verified by taking photographs of the sections of a wire before and after loading where a remarkable change in the surface structure has been revealed. This property of the wires is generally named "Elastic Fatigue".

[†] Hooke's Law is valied within "Elastic Limit" only. Hence under no circumstances should the wire be loaded with more than the maximum possible load and this permissible load is given by half the breaking stress for the wire, since this elastic limit ensues just after the application of this stress.

(*v*) *Place gently half a kilogram weight in the hanger H and wait for about two minutes. This will result in the extension of the wire and consequently the air bubble will get displaced. Bring back the air bubble in its standard position, *i e.*, the centre by raising the screw and note the reading of the micrometer screw. The difference between two readings of the micrometer screw gives the extension produced by half-a-kilogram load.

Now gradually increase the load in equal steps of half-a-kilogram upto the maximum permissible load, adjust the spirit level and note the reading of the micrometer screw each time. Move the screw forward by about one revolution and note its reading again by turning it back till the bubble is in the centre. This reading will be nearly or exactly equal to the previous one depending upon the amount of back-lash present in the screw. The screw should always be moved in *one direction to avoid error due to back-lash.*

(*vi*) Now remove the load in steps of one half-a-kilogram weight till all weights except that used to keep the wire taut have been removed. Take the reading of the micrometer screw after each weight has been removed. The readings of the micrometer screw for various weights taken when the wire is being unloaded should agree closely with those taken when the wire was loaded. If they do not agree, then it means that the experimental wire was initially not free from kinks and hence this set of observations should be rejected. Now load the wire with heavier weight to remove the kinks and repeat the process till a satisfactory set of readings is obtained.

(*vii*) Measure the length of the experimental wire '*b*' from the point of support to the pin-vices, *i.e.*, to the place where the wire is fixed to the rectangular frame.

(*viii*) Now take the mean of the two readings of the micrometer screw for the same load and calculate the mean extension of the wire as given in observation table. The reading for this purpose should be so coupled that no reading is used twice otherwise that reading will become useless when the mean is taken.

(*ix*) Plot a graph between load and extension taking load on the X-axis and extension on Y-axis with suitable scales. It will be a straight line showing that extension is proportional to load which is Hooke's Law. Calculate extension for a definite load from the graph and determine Young's Modulus 'Y'.

---

*For instance, for one particular steel wire the radius was found to be 0·22 cm. Hence the area of cross-section of the wire$=\pi r^2=3{\cdot}14\times 0{\cdot}22)^2=$ 0·00152 cm^2. From the table of constants, breaking stress for steel$=110\times10^8$ dyne/cm^2. Hence breaking load$=0{\cdot}00152\times110\times10^8$ dyne$=16$ kgm (nearly). Therefore load greater than 8 kgm should not be applied to this particular re.

*Observations and Calculations :

Pitch of the screw gauge=1 mm

No. of divisions on the circular scale=100 div.

Least count of the screw gauge= $\frac{1}{100}$=0·01 mm=0·001 cm

Zero correction of the screw gauge=0·0 mm.

Diameter of the Wire :

S. No.	*Reading along any diameter in mm*	*Reading along a perpendicular diameter in mm*	*Mean corrected diameter in cm*
1.	0·64	0·65	0·0644
2.	0·64	0·64	
3.	0·65	0·65	
4.	0 65	0 64	
5.	0·65	0·64	
6.	0·64	0·64	

Radius of the experimental wire 'r'=0·0322 cm

Pitch of the micrometer screw=1·0 mm

No. of division on the head scale=100 div

Least count of the micrometer screw=0·01 mm

Extension of the Wire :

S. No.	Weight in the hanger 'H' in kg	Micrometer Screw readings in mm			Extension for 2 kg in mm	Mean extension (l) for 2 kg in cm
		Loading (x)	Unloading (y)	Mean $\frac{(x+y)}{2}$		
1.	0.0	1.03	1.03	1.03 (a)		0.1275
2.	0.5	1.33	1.34	1.34 (b)	1.27	
3.	1.0	1.65	1.64	1.64 (c)	1.29	
4.	1.5	1.98	1.99	1.98 (d)	1.28	
5.	2.0	2.28	2.32	2.30 (e)	1.26	
6.	2.5	2.62	2.65	2.63 (f)		
7.	3.0	2.94	2.90	2.92 (g)		
8.	3.5	3.24	3.24	3.24 (h)		

---

*Students should carefully note that the reading entered herein were obtained for a particular wire. They should enter here the *actual values* obtained by them in their laboratory.

Length of the wire (L)=259·5 cm

$$\therefore \quad \text{Young's Modulus 'Y'} = \frac{Mg\,.\,L}{\pi r^2\,.\,l}$$

$$= \frac{2\times 1000\times 981\times 259{\cdot}5}{3{\cdot}14\times(0{\cdot}0322)^2\times 0{\cdot}1275} = \frac{2\times 981\times 2595}{314\times(322)^2\times 1275} = 10^{10}$$

$$\log Y = \log 2 + \log 981 + \log 2595 + 10\log 10 - \log 314 - 2\log 322 - \log 1275$$

$$= 0{\cdot}3010 + 2{\cdot}9917 + 3{\cdot}4142 + 10{\cdot}0 - 2{\cdot}4968 - 5{\cdot}0159 - 3{\cdot}1055.$$

$$= 22{\cdot}7069 - 10{\cdot}6182 = 12{\cdot}0887$$

$$\therefore \quad Y = 12{\cdot}27\times 10^{11}\ \text{dyne/cm}^2$$

$$= 12{\cdot}3\times 10^{11}\ \text{dyne/cm}^2$$

Graph :

Average value of extension (l) for 2 kg from the graph
=0·1270 cm.

$$\therefore \qquad Y = \frac{Mg.\,L}{\pi r^2.\,l} = 12{\cdot}3\times 10^{11}\ \text{dyne cm}^{-2}$$

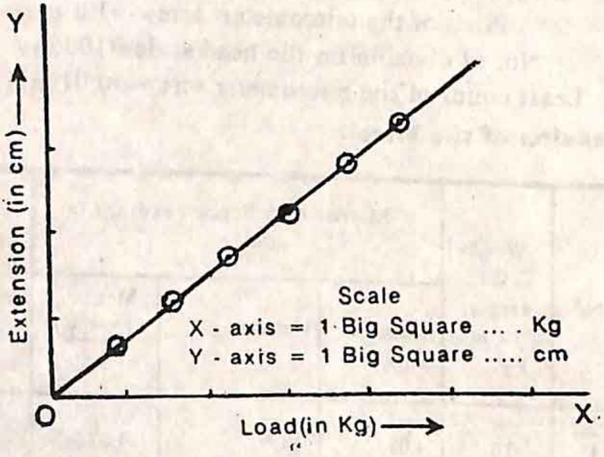


Fig. 33.2.

Result :

Young's Modulus of Elasticity (Y) for the given *copper* wire
$= 12{\cdot}3\times 10^{11}$ dyne cm^{-2}

Actual value from the table
$= 12{\cdot}4\times 10^{11}$ dyne cm^{-2}

$$\text{Error} = \frac{\text{difference}}{\text{Actual value}}\times 100\%$$

$$= 0{\cdot}8\%.$$

Precautions :

1. There should not be any kinks in the wire. It should be loaded heavily at the start to remove the kinks as described in the procedure.

2. The wire should not be loaded beyond elastic limits.

3. The loading or unloading should be in equal step ($\frac{1}{2}$ kg) and should be done carefully and gently.

After each addition or removal of the load, wait for two minutes in order to allow (*i*) the wire to increase or decrease in length, (*ii*) the oscillations to subside and, (*iii*) the wire to attain the room temperature.

4. The radius of the wire should be determined at various points and at each point in two mutually perpendicular directions. This is because a small error in the measurement of '*r*' will result in a large error in the value of Young's Modulus.

5. While calculating the extension from the mean readings of the micrometer screw for various loads, the students are *warned against* taking difference between *the two consecutive readings*. If *a*, *b*, *c*, *d*,......*h* be the mean of loading and unloading the successive difference will be $(b-a)$, $(c-b)$......$(h-g)$ and their mean.

$$\frac{(b-a)+(c-b)+......(h-g)}{7}=\frac{(h-a)}{7}$$

Therefore only first and last readings are utilised and others have been wasted. But if the mean of $(e-a)$, $(f-b)$, $(g-c)$ and $(h-d)$ is taken, all observations are utilised.

6. Before every reading of the micrometer screw, the bubble of the spirit level should be brought at the centre.

7. The micrometer screw should always be rotated in the same direction to avoid back lash error.

8. The length of the wire (L) is the length of the experimental wire '*b*', and *not* that of the reference wire '*a*'.

9. The graph should be smooth and should pass through nearby all the points.

Sources of Error :

(*i*) The weights used may not be standard weights.

(*ii*) The wire may not be of uniform area of cross-section throughout its length.

(*iii*) The radius of the wire may change while loading or unloading.

ORAL QUESTIONS

Q. 1. *What is elasticity ?*

Ans. All material bodies when subjected to suitable forces, suffer a change either in size or in shape or in both and get deformed. When the deforming force is removed the body tends to come back to its original condition. This property of a body by virtue of which it regains its original condition after the removal of the deforming force is called elasticity.

Q. 2. *What are (i) Elastic bodies, and (ii) Plastic bodies ?*

Ans. Bodies which recover completely after the removal of deforming forces are called perfectly elastic bodies and those which do not show any tendency to recover are called perfectly plastic bodies.

In actual practice, we have no perfectly elastic or perfectly plastic bodies, the difference being that of a degree.

Q. 3. *What is stress ? When is stress called (i) compressive, (ii) tensile ?*

Ans. The deforming forces applied to a body give rise to forces of reaction inside it, tending to restore it back to its original condition. This restoring force set up inside the body measured per unit area is called stress. It is equal in magnitude but opposite in direction to the applied deforming force per unit area provided the elastic limit is not exceeded.

If the deforming force is trying to press the body, the stress is named compressive stress but if the deforming force be of a nature of pull or tension, the stress is called tensile stress.

Q. 4. *What are the units of stress ?*

Ans. Its units are dynes/cm^2 in C.G.S. system ; newton/metre^2 in M.K.S. system and S.I. system.

Q. 5. *What are the dimensions of stress ?*

Ans. Its dimensions are $ML^{-1}T^{-2}$.

Q. 6. *What are the different types of stress ? Define them.*

Ans. Stress is of three kinds :

(*i*) Longitudinal stress......it is force per unit area which produces a change in length.

(*ii*) Volumetric stress......it is force per unit area which produces a change in volume.

(*iii*) Shearing stress......it is force per unit area which produces a change in the shape of the body.

Q. 7. *What is strain ? Mention the units of strain.*

Ans. Due to the deforming force, there is a change in the dimensions of the body. Strain is defined as the ratio of the change in configuration to the original configuration (*i.e.*, original length or volume).

Since it is a ratio, it has no units.

Q. 8. *What are the three types of strain ? Define them.*

Ans. (*i*) *Longitudinal Strain*—it is change in length per unit length.

(*ii*) *Volumetric Strain*—it is change in volume per unit volume.

(*iii*) *Shearing Strain*—it is change in shape due to relative movement of some plane of the body with respect to other. It is measured by the angle through which a side is turned.

Q. 9. *Which comes first—stress or strain ?*

Ans. Stress depends upon strain, *i.e*, strain is an independent quantity. This is because unless deformation is produced there is no restoring force set up inside the body.

Q. 10. *What is elastic limit ?*

Ans. The limit upto which extension is proportional to the tension and the body regains its original position after the deforming forces have been removed, is called elastic limit.

Q. 11. *What is yield point ?*

Ans. The yield point is the point at which the wire begins to flow, thinning uniformly even without any increase in the load.

Q. 12. *What is breaking point ?*

Ans. After yield point, a stage is reached when the wire goes on increasing in length and ultimately breaks. The point where it breaks is called breaking point.

Q. 13. *What is breaking stress ?*

Ans. The minimum load with which the wire breaks is called breaking load. Breaking load per unit area is called breaking stress.

Q. 14. *What is stress—strain curve ?*

Ans. Suppose a wire of uniform area of cross-section is suspended from a rigid support and is stretched by hanging a load at the other end. If the load is increased gradually, the length of the wire also increases accordingly. If the values of stress and corresponding strain are determined at various stages and a graph is plotted between them, then the stress-strain, curve is obtained as shown in Fig. 32.3.

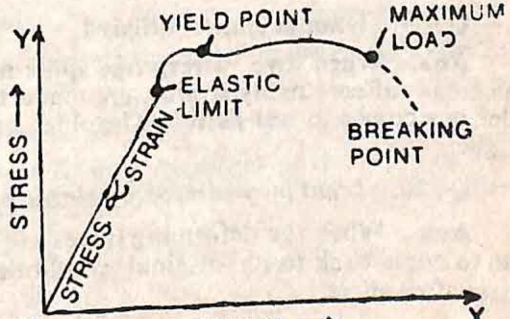


Fig. 33.3

Q. 15. *What is Hooke's Law ? Who stated Hooke's Law ?*

Ans. According to Hooke's Law, stress is proportional to the strain, within the elastic limits.

Robert Hooke stated this law.

Q. 16. *What is Young's Modulus of Elasticity ? What are its units ?*

Ans. Young's Modulus (Y)$=\dfrac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}}$

$$=\frac{\frac{F}{a}}{\frac{l}{L}} \quad \text{where } F=\text{force applied}$$

a=area ; l=change in length ;

L=original length

Units :

C.G.S. → dyne/cm^2

M.K.S. → N/m^2

Q. 17. *What is meant by tensile strength ?*

Ans. It is the value of breaking stress per sq. mm area of cross-section.

Q. 18. *What is permanent set ?*

Ans. If we increase the load beyond elastic limit and then the deforming forces are removed, the wire does not regain its original position but attains a permanent extension. This is called permanent set.

Q. 19. *What is elastic fatigue ?*

Ans. When two wires, one quite new and other an old one which has suffered many strains, are made to oscillate together, the older one comes to rest earlier. The older wire is said to have elastic fatigue.

Q. 20. *What do you mean by elastic after-effect ?*

Ans. When the deforming forces are removed, the body takes time to come back to its original condition. This delay is called elastic after-effect.

Q. 21. *What is the effect of temperature on the modulus of elasticity ?*

Ans. The value of modulus of elasticity decreases with rise of temperature.

Q. 22. *Are there metals whose elastic properties do not change with temperature ?*

Ans. Yes ; these metals are called Elinvars which are alloys of nickel and steel. Their elasticity does not change.

Q. 23. *Is there any practical application of the knowledge of elasticity ?*

Ans. Yes. With its knowledge we can calculate the force which a girder or the piston rod of steam engine, etc., can withstand.

Q. 24. *Which is more elastic—steel or rubber ?*

Ans. If equal forces are applied to steel and rubber, less strain is produced in steel than rubber. Therefore steel is more elastic than rubber.

Q. 25. *Why do we use long wires ?*

Ans. So that the extensions may be large and may be measured accurately.

Q. 26. *Why do we use two similar wires ?*

Ans. To avoid error due to yielding of the support and increase in length due to change in room temperature.

Q. 27. *Why do we wait before taking reading after each loading and unloading ?*

Ans. So that the wire may cool down to its original temperature and to allow the wire to increase or decrease in length.

Q. 28. *On what factors 'Y' depends ?*

Ans. It depends upon (*i*) the nature of the material of the body, (*ii*) form in which the body is taken and, (*iii*) temperature of the body.

Q. 29. *Does 'Y' depend on the diameter of the wire ?*

Ans. No ; it does not depend upon the diameter. It depends on nature of the material.

Q. 30. *Which measurement should be most accurate ?*

Ans. Change in length (l) and the radius of the wire (r). A small error in the measurement of 'r' results in a large error in the value of 'Y' because $Y=\frac{Mg.L}{\pi r^2.l}$

Q. 31. *What is Poisson's ratio ?*

Ans. Poisson's ratio is defined as the ratio of lateral contraction to the longitudinal extension when a wire is stretched in a linear direction. If L and D be the initial length and diameter respectively and l and d be the increase in length and decrease in diameter, then

$$\text{Poisson's ratio } \sigma = \frac{\frac{d}{D}}{\frac{l}{L}} = \frac{L\,.\,d}{l\,.\,D}.$$

Since it is a ratio of two similar quantities, it has no units and no dimensions.

Q. 32 *Why is it advised to rotate the screw in the same direction ?*

Ans. To avoid backlash error.

Q. 33. *Is there any change in the value of 'Y' if a larger or thicker wire is used ?*

Ans. Yes. Young's Modulus of a wire is directly proportional to the length of the wire and inversely proportional to the area of cross-section of the wire.

Experiment 34 :

To study the variation in length of a rubber band with applied force (beyond elastic limit).

Apparatus :

Rigid support, Peg or nail, Thick rubber string, Pan, Slotted weights of 50 gm each, a fine pointer, Vertical scale in mm.

Theory :

If a rubber string of length 'L' has a load of 'Mg' suspended from its lower end, then a force equal to Mg dynes acts vertically along the axis of the string and consequently the rubber string extends in length. Let this extension be 'l' cm. Then the longitudinal strain produced in the rubber string is $\frac{l}{L}$. If the area of cross-section of the rubber string be 'A', then the normal stress set up in the string in the equilibrium state is given by $\frac{Mg}{A}$. The Young's Modulus (Y) for the rubber string is given by

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{\frac{Mg}{A}}{\frac{l}{L}} = \frac{Mg\,.\,L}{Al}.$$

If the radius of the rubber string be 'r' then $A=\pi r^2$.

Hence

$$\boxed{Y=\frac{Mg\,L}{\pi r^2 . l}}$$

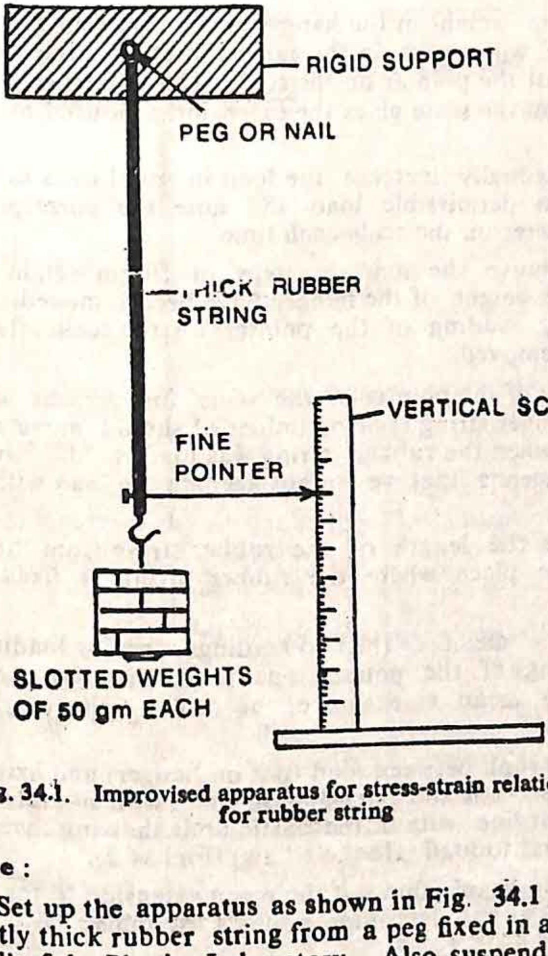


Fig. 34.1. Improvised apparatus for stress-strain relationship for rubber string

Procedure :

1. Set up the apparatus as shown in Fig. 34.1 *i.e.*, suspend a sufficiently thick rubber string from a peg fixed in a rigid support on the wall of the Physics Laboratory. Also suspend the hanger of the slotted weights to the other end of the rubber string. Attach a fine pointer near the lower end of the rubber string which moves over a vertical scale. See that the tip of the pointer is on the scale.

2. Find the diameter of the rubber string with the screw gauge at six different places when the hanger is not suspended. At each point, measure the diameter in two mutually perpendicular

directions. While measuring the diameter, the string should not be pressed too hard.

3. Find the least count of the scale and suspend the hanger of the slotted weights (the hanger itself weighs 50 gm) gently and wait for some time so that the rubber string attains the full extension. Now read the position of the pointer on the vertical scale.

Place a 50 gm weight in the hanger gently and wait for about 2 minutes. This will result in the extension of the rubber string. Note the reading of the pointer on the scale. The difference between the two readings on the scale gives the extension produced by 50 gm load.

4. Now gradually increase the load in equal steps of 50 gm upto the maximum permissible load and note the corresponding reading of the pointer on the scale each time.

5. Now remove the load in steps of 50 gm weight till all weights except the weight of the hanger have been removed. Take the corresponding reading of the pointer on the scale after each weight has been removed.

The readings of the pointer on the scale for various weights taken when the rubber string is being unloaded should agree closely with those taken when the rubber string was loaded. If *they do not agree then it means that we are not keeping the load within the elastic limit.

6. Measure the length of the rubber string from the point of support to the place where the rubber string is fixed to the hanger.

7. Take the mean of the two readings (one for loading and other for unloading) of the pointer on the scale for the same load and calculate the mean extension of the rubber string as given in observation table.

8. Plot a graph between load (put on hanger) and extension, taking load on the x-axis and extension on y-axis with suitable scales. It will be a straight line within the elastic limit showing that extension is proportional to load (Hooke's Law) (Fig. 34.2).

9. From the graph, find out the mean extension 'l' for a definite load (say 100 g) and determine Young's Modulus.

Observations and Calculations :

Pitch of the screw gauge=……mm.

No. of divisions on the circular scale=……

Least count of the screw gauge=……mm=……cm.

Zero correction of the screw gauge=……mm

---

*Particularly the zero weight in the hanger readings.

Diameter of the Rubber String :

S. No.	*Reading along any diameter in cm*	*Reading along a perpendicular diameter in cm*	*Mean corrected diameter (d) in cm*
1.			
2.			
3.			
4.			
5.			
6.			

$$\text{Mean radius } (r)=\frac{d}{2}=\text{......cm}$$

Length of the rubber string (*i*) ......cm, (*ii*) ......cm, (*iii*) ......cm

Mean length of the rubber string$=$L$=$......cm

Extension of the Rubber String :

S. No.	*Weight in the hanger in g*	*Reading of the pointer on the vertical scale in cm*		
		Loading (*x*)	*Unloading* (*y*)	*Mean* $\left(\frac{x+y}{2}\right)$
1.				
2.				
3.				
4.				
5.				
6.				

Determine the average value of extension '*l*' for 100 g from the load-extension graph.

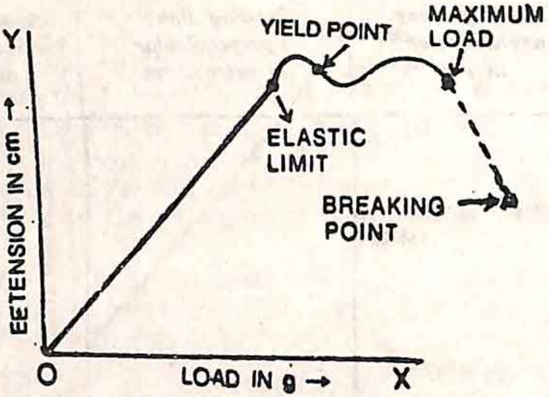


Fig. 34.2. Stress-strain curve.

Substitute the values of M, L, *l* and *r* in formula

$$Y=\frac{Mg.L}{\pi r^2.l}$$ and calculate the value of 'Y'.

Result :

The Young's Modulus for the rubber string

=.........dynes/cm²

Precautions :

1. The rubber string should not be loaded beyond elastic limits.

2. The loading and unloading should be done in equal steps (50 gm) and it should be done gently and carefully.

3. After each addition or removal of the load, wait for some time in order to allow the string to increase or decrease in length.

4. The radius of the rubber string should be determined carefully at various points and at each point in two mutually perpendicular directions.

This is because a small error in the measurement of '*r*' will result in a large error in the value of 'Y'.

5. While measuring the radius, the string should not be pressed too hard.

6. The rubber string used should be sufficiently thick so that the extension produced by 50 g wt. is not very large.

Sources of Error :

(*i*) Yielding of support and change in room temperature.
(*ii*) Change in the area of cross-section
(*iii*) Weights used may not be standard weights.

ORAL QUESTIONS

[Same as in experiment 32.]

Experiment 35 :

To find the spring constant of a helical spring by measuring time period of vertical oscillations of a known load and check by measuring its extension by a known force.

Apparatus :

A helical spring, metre scale, two clamps and stands, stop-watch, light pointer for spring, hanger with slotted weights, 1 kg. weight.

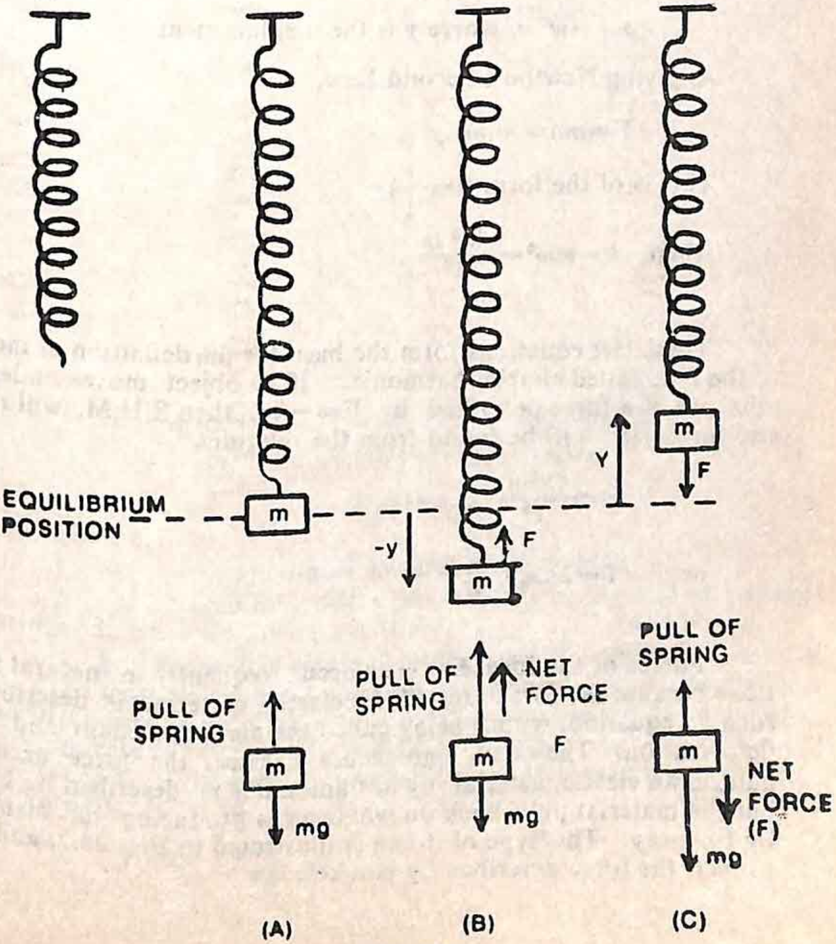


Fig. 35.1.

Theory :

If a mass is oscillating back and forth with simple harmonic motion, there must be a force acting. For example, you hang a mass carefully on a spring, letting it hang at rest. If the mass is pulled down as in Fig 35.1, the spring pulls it back toward its equilibrium position. If the mass is lifted slightly, gravity tends to pull it down. Whenever the mass is displaced, there is a force pulling it back to the equilibrium position. If the displacement is up (positive), the restoring force is down (negative). If the displacement is down (negative), the restoring force is upward (positive). The restoring force is opposite in sign to the displacement.

The force is related to the acceleration by Newton's Second Law, $F=ma$. In simple harmonic motion, the acceleration is described by

$a=-w^2\,y$, where y is the displacement

Applying Newton's Second Law,

$$F=ma=-mw^2y$$

This is of the form $F=-ky$

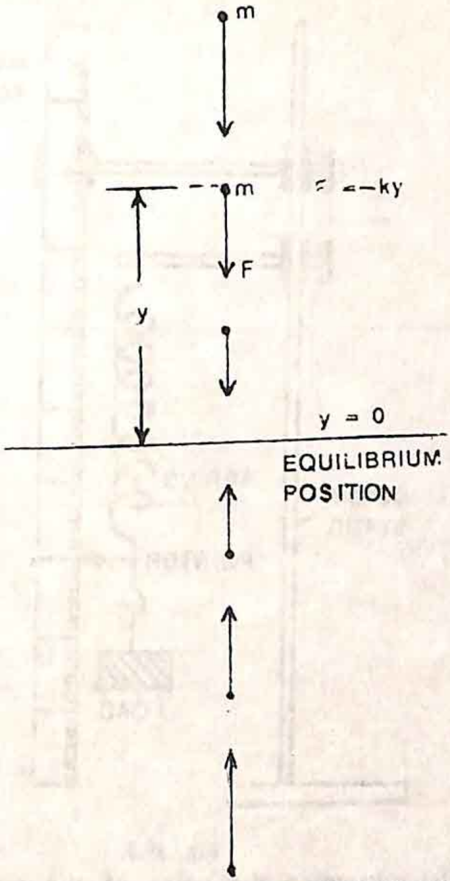
where $k=m\omega^2=\dfrac{4\pi^2\,m}{T^2}$.

These last equations form the basis for the definition of motion of the type called simple harmonic. If an object moves under the influence of a force described by $F=-ky$, then S.H.M. will result and the period will be found from the relations

$$k=\frac{4\pi^2m}{T^2}$$

or $$T=2\pi\sqrt{\frac{m}{k}}$$

Forces of the form $F=-ky$ occur frequently in natural situations because the force exerted by elastic materials is described by such an equation, with k being called the elastic constant and y the deformation. The minus sign occurs because the force needed to deform an elastic material by an amount y is described by $F=ky$, but the material pulls back on whatever is producing the distortion by $F=-ky$. This type of force is illustrated in Fig. 35.2, and it is, in fact, the force described by Hooke's law.



A REPRESENTATION OF A FORCE DESCRIBED BY F = -KY, THE TYPE THAT RESULTS IN S H.M

Fig. 35.2.

Procedure :

1. Set up the apparatus as shown in Fig. 35.3.

2. Attach a suitable mass (m) say 1 kg at one end of a helical spring. Suspend the spring vertically.

3. Pull the weight down through a small distance and let it go. The mass suspended by the lower end of the spring would start executing vertical oscillations. Find the time for say 10 oscillations with the help of stop-watch and then calculate its time-period (T) and hence find the spring constant (k) of the helical spring from the formula

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{or} \quad k = \frac{4\pi^2 m}{T^2}$$

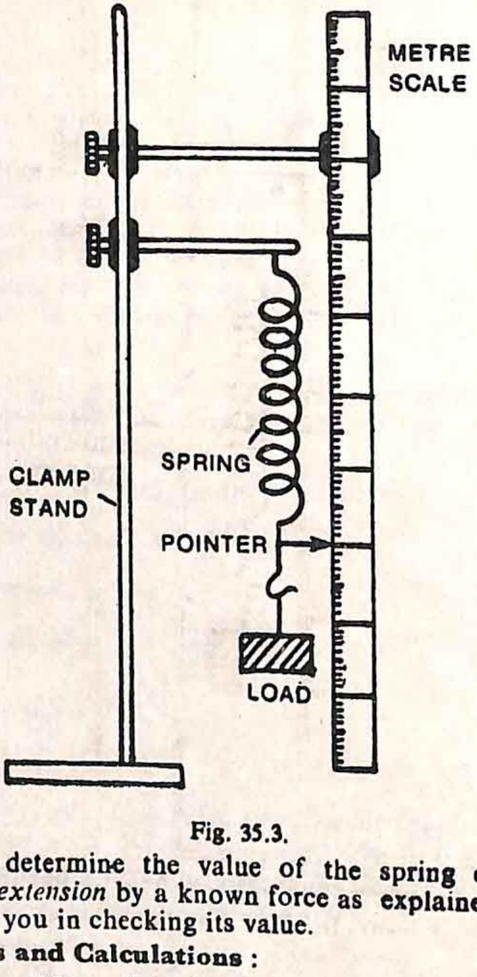


Fig. 35.3.

4. Also determine the value of the spring constant (*k*) by measuring *its extension* by a known force as explained in Expt. 32. This will help you in checking its value.

Observations and Calculations :

Mass of the load $= m =$ ......kg

Least count of the stop-watch $=$ ......sec

S. No.	*No. of vertical oscillations (n)*	*Time (t) for n vertical oscillations in Sec*			*Time Period* $T=\frac{t}{n}$ *in Sec*
		1	*2*	*Mean*	

$$\text{Spring Constant } (k) = \frac{4\pi^2 m}{T^2}$$

$$= \ldots\ldots \text{Nm}^{-1}$$

Verification :

S. No.	*Force (F) in N*	*Pointer reading in mm*			*Extension (e) in mm*	*Spring constant* $K=\frac{F}{e}$ *(in* Nm^{-1}*)*
		Loading	*Unloading*	*Mean*		
					Mean Value=	

Result :

Spring constant (k)
of the given helical spring$=\ldots\ldots \text{Nm}^{-1}$

Precautions :

[Same as in Expt. 32.]

Precautions :

(1) to (4) same as in Expt. 32.

(5) While oscillating the spring see that the amplitude of oscillations is *small* and the spring oscillates in a *vertical plane.*

(6) Note the times very carefully and for this purpose employ an accurate stop-watch. The time periods occur *squared* in the formula and hence any error committed in their evaluation will introduce double the error in the results.

Sources of Error :

(Same as in Expt. 32).

ORAL QUESTIONS

(Same as in Expt. 32.)

SECTION H

Experiment 36 :

(a) *To study the relation between length and tension for constant frequency of a stretched using a sonometer.*

(b) *Determine the frequency of a tuning fork.*

Apparatus :

A sonometer, the given tuning fork, a rubber pad, sensitive physical balance, weight box, solid kilogram weights, paper rider.

Theory :

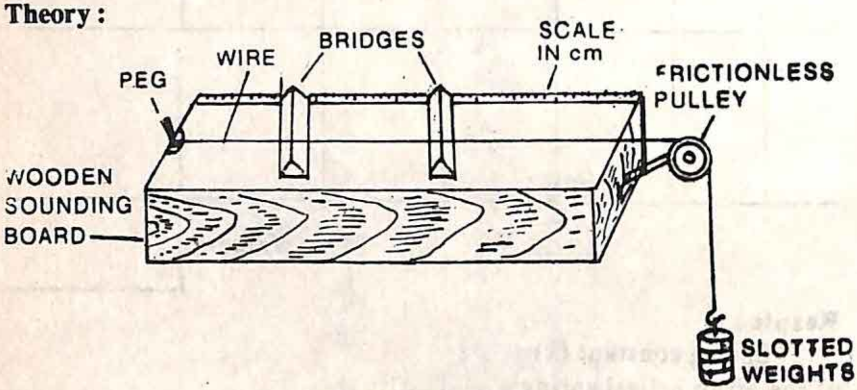


Fig. 36.1. Sonometer

When the wire of the sonometer (Fig. 36.1) having certain tension is plucked in the middle, it is thrown into stationary vibration having nodes at the two edges of the bridges and an antinode in the middle. The volume of the note emitted by the wire is considerably increased with the help of the sounding board due to the forced vibrations imposed by the vibrating wire upon it and the air contained therein.

When the note emitted by the sounding length of the wire is in unison with the tuning fork, the frequency '*n*' of the later is given by

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{\lambda}\sqrt{\frac{T}{m}}$$

where λ = wave length

l = length of the vibrating segment of the sonometer wire

T=Tension applied to the wire

=Mg where M is the total mass suspended from the wire.

m = mass of the wire *per unit length.*

Procedure :

(1) Check that the pulley of the sonometer is frictionless and

then stretch the sonometer wire with a suitable known tension by placing weights on the hanger. Sound the tuning fork by gently striking it at the end of a prong against a soft rubber pad and place it on the top of the sounding box, A loud sound will be heard. Now starting with a small length of the wire between the bridge, *pluck in the middle so as to excite its fundamental mode of vibration and compare its note with that of the tuning fork.

(2) Now shift the position of one of the bridges so as to increase the length of the sounding wire till the frequency of the tuning fork is very nearly equal to that of the wire. This will be apparent from the appearance of beats, when the two are sounded together. Shift the movable bridge slowly till the beats appear to be drawn out, meaning thereby that the number of beats diminishes and the frequency of the wire approaches that of the tuning fork. Finally move the bridge further till the beats disappear and the two are in unison.

If in this position a small paper rider (V-shape) be placed on the middle of the wire, it will be energetically thrown off.

(3) Now measure carefully the resonant length of the wire between the bridges on the scale.

(4) Repeat the experiment twice for the same load. Take three sets of observations for *three different loads.*

(In the process of increasing the load on the hanger, be careful not to stretch the wire beyond the elastic limit.)

(5) Next cut off a length (preferably one metre) of the wire, weigh it in a sensitive physical balance and thus determine its mass per unit length and then calculate '*n*' from the formula given above.

Observations :

S. No.	Load applied (*M*) kg	Tension $T=Mg$	Length of the wire (between the bridges) in unison				Wavelength $\lambda=2l$	$\frac{\sqrt{Mg}}{\lambda}$	l^2	Mean l^2	$\frac{Mg}{l^2}$	$n=\frac{1}{2l}\sqrt{\frac{Mg}{m}}$ (*Hz*)
			(1) *cm*	(2) cm	(3) *cm*	*Mean* (*l*) *cm*						
1.												
2.												
3.												
4.												

Mean n= ......*Hz*

---

*Avoid the use of finger nails in this process.

Calculations :

Length of the wire $=a=$......cm

Mass of length 'a' cm of wire$=x=$......g

$$\text{Mass per unit length}=m=\frac{x}{a}=\text{......g/cm}$$

$$\text{Now}\quad n=\frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$=\frac{1}{2l}\sqrt{\frac{Mg}{m}}$$

where g=Acceleration due to gravity at a place.

$$\text{Hence } n^2=\frac{g}{4m}\,\frac{M}{l^2}=\text{........}$$

$\therefore$ $n=$............Vibs./sec (or cycles/sec) or hertz.

Note : We can also calculate 'n' from the graph between T (=Mg) and l^2, which is a straight line. From this graph read off T and l^2 for any point lying on it and calculate the frequency 'n'.

Result :

(*a*) Since $\frac{\sqrt{Mg}}{\lambda}$=constant, therefore $\frac{\sqrt{\text{Tension}}}{\text{Wavelength}}$=constant

(*b*) the frequency of the tuning fork

(*i*) as determined experimentally=............Hz

(*ii*) as given on the fork itself=............Hz

$\therefore$ Error=............Hz=............%

Precautions :

1. The sonometer wire should be uniform and free from kinks.

2. For bringing the wire in unison with the tuning fork, start with a small length of the wire and alter the length in small increments.

3. Unison should always be tested by the method of removal of beats.

4. While finding out the tension of the wire, do not forget to add the mass of the hanger.

5. While increasing the tension of the wire, be careful that the wire is not stretched beyond elastic limit. For this purpose, before starting the experiment have an idea of the magnitude of the breaking load of the given wire with the help of Table of Physical constants.

6. The excited tuning fork should be placed vertically with its stem pressed against the top of the sounding box.

7. The tuning fork should never be struck against a hard surface and when excited should be held in hand with its stem.

8. After completing the experiment remove weights from the hanger. Under no circumstances should the wire be left in a stretched condition.

9. In plucking a string to excite its note, care should be taken to avoid touching the string with the finger nail as this may introduce overtones. The string should be pulled aside between the thumb and the finger.

Sources of Error :

1. In the derivation of the formula $n=\frac{1}{2l}\sqrt{\frac{T}{m}}$ it has been assumed that the wire is perfectly flexible. Hence due to the rigidity of the experimental wire, an error shall creep in the result.

2. If the wire is not uniform or if its composition is variable, then also the result will be erroneous.

3. In this horizontal pattern of the sonometer, there is always present some friction at the pulley ; hence the value of tension is less than that actually applied. This consequently affects the value of the frequency. Moreover the tension on the two sides of the bridges may not be the same.

4. There is always some practical difficulty for the ear (specially when it is untrained) to establish perfect unison in two musical sounds.

ORAL QUESTIONS

Q. 1. *How is sound produced* ?

Ans. Sound is produced due to vibration of a body.

Q. 2. *Can sound travel in vacuum* ?

Ans. Sound cannot travel in vacuum. A material medium is necessary for its propagation.

Q. 3. *What is a wave motion* ?

Ans. Wave motion is the disturbance which travels in a material medium and is due to the repeated motion of the particles of the medium about their mean position ; the motion being handed over from one particle to the next.

Q. 4. *Define S.H. Motion* ; *Vibration* ; *Time Period* ; *Frequency* ; *Phase* ; *Phase difference and Amplitude.*

Simple Harmonic Motion (*S.H.M.*) : A particle is said to be executing S.H.M., if its acceleration is always proportional to its distance from some fixed point in its path and is always directed towards that point.

Vibration. One vibration is the to and fro motion between two consecutive passages of the particle in the same direction.

Time Period (*T*). The time taken to complete one vibration is called the time period.

Frequency (*n*). The number of vibrations completed by the vibrating particle in one second is called the frequency.

Phase. The phase of a vibrating particle at any instant is its state as regards its position and direction of motion at that instant. It is measured either in terms of angle that the particle has described or the time that has elapsed (measured as the fraction of time period T), since the particle last passed through its normal position in the positive direction.

Phase difference. Phase difference between two particles indicates the extent by which the two particles are out of step with each other. It, too, is measured by the time (denoted as a fraction of T) by which ore particle is ahead of the other.

Amplitude. The maximum distance covered by the vibrating particle on either side of its mean position is called is amplitude (Fig. 36.2)

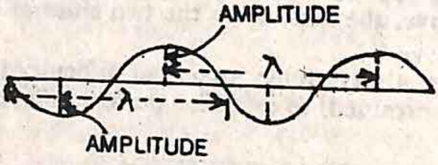


Fig. 36.2

Q. 5. *What are the types of Wave Motion ?*

Ans. (1) Transverse wave motion.

(2) Longitudinal wave motion.

Q. 6. *What is a transverse wave ?*

Ans. It is a wave in which the particles of the medium in which the wave propagates, vibrate along a line which is perpendicular to the direction of propagation of the wave.

Sound travels in the form of transverse waves in strings. Light is a Transverse Wave.

Q. 7. *What is a longitudinal wave ?*

Ans. A wave motion in which the individual particles of the Medium vibrate back and forth along the direction of propagation of the wave is called a longitudinal wave motion.

Q. 8. *Define wave length.*

Ans. It is the distance travelled by the wave during the time the vibrating particle or any other particle of the medium of propagation completes one vibration. It is denoted by the letter λ (Fig. 36.2).

Q. 9. *Can transverse waves be produced in air ?*

Ans. No, because air does not possess elasticity of shape.

Q. 10. *What types of waves are sound waves ?*

Ans. Sound waves are longitudinal waves.

Q. 11. *What is the relation between frequency and time period ?*

Ans. The reciprocal of time period is called frequency.

$$\boxed{n=\frac{1}{T}}$$

Q. 12. *Why is the tuning fork so-called ?*

Ans. Because it produces a musical sound when struck and has the shape of a fork.

Q. 13. *Why the tuning fork has two prongs ?*

Ans. Firstly in order to have a node between the two antinodes which are produced at its ends. Secondly if there is only one prong the vibration will die out as soon as the stem is touched.

Thirdly the two prongs reinforce each other and maintain vibrations for a longer time.

Q. 14. *What are the factors on which frequency of a tuning fork depend ?*

Ans. It depends upon the length, thickness and the material of the prongs.

It is inversely proportional to the square of the length of the prongs. It is directly proportional to the thickness of the prong.

Q. 15. *Of what material the tuning forks are made ?*

Ans. Generally these are made of steel because its elasticity is high and density low. This is because the frequency of a tuning fork is inversely proportional to the square root of the density of the material of the fork and directly proportional to the square root of the elasticity of the material of the fork.

Q. 16. *What is the effect of temperature on the frequency of a fork ?*

Ans. The increase in temperature increases the length of the prongs and thus frequency is decreased.

Q. 17. *How is the effect of temperature minimised ?*

Ans. By making the tuning fork of an alloy called ELINVAR, which is made of steel, nickel and chromium. It has a very low temperature coefficient.

Q. 18. *Why do we strike the tuning fork on the rubber pad ?*

Ans. To strike it gently so that only fundamental note may be produced.

Q. 19. *What is the effect of loading the prongs with wax ?*

Ans. The frequency of the fork decreases.

Q. 20. *What is the effect of filing the prongs ?*

Ans. The frequency of the fork increases.

Q. 21. *Why do the vibrations of the tuning fork stop after sometime ?*

Ans. Due to frictional force of air.

Q. 22. *What is the relationship between velocity of wave motion frequency and wave length ?*

Ans. $V=n\lambda$ where V=Velocity of sound

n=frequency

and λ=wavelength

Q. 23. *What type of vibrations are produced in prongs and the stem of the tuning fork ?*

Ans. The free ends of the fork perform transverse vibrations while the longitudinal waves are produced in the stem.

Q. 24. *What happens if we strike the tuning fork with a great force ?*

Ans. It will not give pure note but overtones may be produced.

Q. 25. *Why does the fork remain vibrating even when we hold it by the handle ?*

Ans. Because the handle performs longitudinal vibrations which are not stopped by touching.

Q. 26. *Why does the tuning fork give a pure note only ?*

Ans. Because the first overtone is very weak. It is about six ctaves higher than the fundamental note.

Q. 27. *Why does the tuning fork stop vibrating when we touch the prong ?*

Ans. Because the prongs execute transverse vibrations which stop on touching.

Q. 28. *Does the frequency of the tuning fork change with time ?*

Ans. No ; it remains same. Only the decrease in amplitude decreases the loudness of the sound because the intensity of sound is directly proportional to the square of the amplitude.

Q. 29. *What is the difference between a note and a tone ?*

Ans. A note is a sound of definite frequency produced by a periodic complex vibrations where as a sound produced by a pure sine vibrations is called a tone.

Q. 30. *What is a fundamental note ?*

Ans. The note produced of lowest frequency is called a fundamental note.

Q. 31. *What are overtones ?*

Ans. The notes of the higher frequencies other than fundamental note are called *overtones.*

Q. 32. *What is the difference between frequency and pitch ?*

Ans. Frequency is the number of vibrations made per second. Pitch is the physical characteristic of sound depending upon frequency. The greater the frequency, the higher the pitch and vice-versa.

Q. 33. *How do the prongs vibrate even though we strike only one ?*

Ans. Energy is transmitted from one prong to another through the material of the fork and thus both begin to vibrate.

Q. 34. *Why is sonometer so called ?*

Ans. Because it enables us to measure the frequency of vibrating body producing sound.

Q. 35. *Why are there holes in the wooden box ?*

Ans. So that internal air and external air may have a direct contact.

Q. 36. *Why is the wooden box hollow ?*

Ans. So that the whole of the air inside the box may begin to resonate thus increasing the intensity of the note.

Q. 37. *What types of waves are produced in the sonometer wire ?*

Ans. Transverse and stationary waves are produced in the wire.

Q. 38. *What types of waves are produced in the surrounding air ?*

Ans. Longitudinal and progressive waves are produced in the surrounding air.

Q. 39. *What type of wire should we use ?*

Ans. The wire should be thin, uniform and flexible.

Q. 40. *What are progressive waves ?*

Ans. The progressive waves are those in which all the particles execute simple harmonic motion and there is a regular phase difference between the particles of the medium.

Q. 41. *What are stationary waves ?*

Ans. When two sets of progressive waves, having the same amplitude and period, but travelling in opposite directions with the same velocity meet each other in a confined space, the result of their superposition is a set of waves, which only expand and shrink but do not proceed in either direction. These waves are called stationary waves.

Q. 42. *Why are they called stationary waves ?*

Ans. They are so called because they remain confined in the region in which they are produced and are non-progressive in character.

Q. 43. *What are nodes ?*

Ans. Nodes are those points in the stationary waves where the particles are permanently at rest and strain is maximum.

Q. 44. *What are antinodes ?*

Ans. Antinodes are those points in the stationary waves where the displacement is maximum and strain is zero.

Q. 45. *What is the distance between two consecutive nodes or antinodes ?*

Ans. It is equal to half the wave length ($\lambda/2$).

Q. 46. *What is the distance between a node and an antinode ?*

Ans. It is equal to quarter of the wave length ($\lambda/4$).

Q. 47. *What is the function of knife edges or bridges ?*

Ans. The knife edges (or bridges) reflect the sound and produce stationary waves.

Q. 48. *Is it necessary to keep the wire horizontal ?*

Ans. No, it can be kept vertical also.

Q. 49. *How is the energy communicated from wire to the sound box ?*

Ans. Through the bridges.

Q. 50. *What are the laws of vibration of strings ?*

Ans. The fundamental frequency of vibrating string is inversely proportional to the length ; directly proportional to the square root of the tension and inversely proportional to the square root of the mass per unit length.

Q. 51. *How is the frequency of the wire affected if the wire is made hollow ?*

Ans. Its frequency increases because mass per unit length decreases.

Q. 52. *What is the function of a V-shaped paper rider ?*

Ans. The paper rider placed at the middle point (antinode) is thrown off when the resonance is obtained.

Q. 53. *What do you mean by unison ?*

Ans. The two notes are said to be in unison if they have same frequency.

Q. 54. *Why do you start from the minimum length of the wire ?*

Ans. So that the wire vibrates in one segment and we may get the position of the fundamental note.

Q. 55. *Can we use a rubber wire in a sonometer ?*

Ans. No, since it is not rigid. The vibrations die out quickly.

Q. 56. *Why is the sound so loud when we press the stem of the vibrating tuning fork against the board ?*

Ans. Due to forced vibrations produced in the sounding board which has a large surface area, so greater is the loundness.

Q. 57. *What do you mean by sympathetic vibrations (or Resonant vibrations) and Resonance.*

Ans. Sympathetic (or Resonant) vibrations are those which are produced in a body due to the presence of a neighbouring vibrating body of the same frequency. The phenomenon is called Resonance.

Q. 58. *What are beats ?*

Ans. The alternate waxing and waning of sound when the notes of nearly same frequency are sounded together is known as phenomenon of beats.

Q. 59. *What are the limits of audibility ?*

Ans. We can hear sounds between the frequencies varying from 20 to 20,000 cycles per second.

Q. 60. *What will be the nature of the graph between 'n' and 'l' ?*

Ans. Hyperbola.

Q. 61. *What type of graphs will be between (a) n and $\frac{1}{l}$ (b) n^2 and T (c) l^2 and T ?*

Ans. Straight lines.

Q. 62. *What type of graph do you expect between n and T ?*

Ans. A parabola.

Q. 63. *How will the length between the bridges change if*

(*a*) *tension is doubled.*

(*b*) *A tuning fork of double the frequency is used.*

(*c*) *radius of the wire is doubled.*

Ans. (*a*) The length will become $\sqrt{2}$ times the first length.

(*b*) and (*c*). The length will become one-half.

Q. 64. *What are the uses of a sonometer ?*

Ans. A sonometer is used

(*a*) to determine the frequency of a tuning fork.

(*b*) to determine the weight of a given body.

(*c*) to verify the laws of vibrating strings, etc.

Q. 65. *Draw a diagram showing* **nodes** *and* **antinodes** *on a vibrating tuning fork.*

Ans.

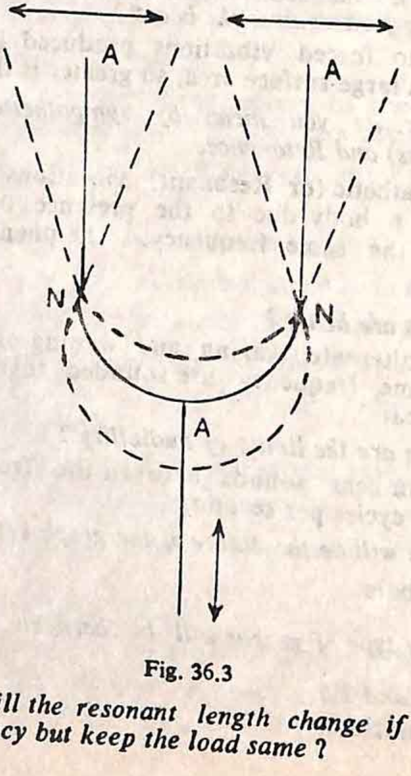


Fig. 36.3

Q. 66. *Will the resonant length change if we have a fork of different frequency but keep the load same ?*

Ans. Yes ; the resonant length will change as the frequency is inversely proportional to the resonant length of the wire keeping tension constant *i.e.*,

$\boxed{n_1 l_1 = n_2 l_2}$ provided the load is the same.

Q. 67. *How does the friction at the pulley affect the result ?*

Ans. Due to friction the value of tension (T) is less than that actually applied. This consequently affects the value of the frequency (n) as $n \propto \sqrt{T}$.

Experiment 37 :

To study the relation between frequency and length of a stretched string using a sonometer.

Apparatus :

A sonometer, wire, hanger, half kilogram weights, two wedges, wooden blocks, 3 tuning forks of different frequencies, a pair of scissors, sensitive balance, a weight box, a metre rod, and a rubber hammer.

Theory :

(Same as in Expt. 36).

Procedure :

Proceed as in **Experiment 36** and keeping the same tension, find the length of the wire which vibrates in resonance with each of the tuning forks.

Observations and Calculations :

Load including the hanger=............kg

S. No.	*Frequency* (n) *Hz*	*Distance between wedges (in cm)*			$n \times l$
		Length increasing	*Length decreasing*	*Mean* (l)	
1.					
2.					
3.					

Verification : Since $n \times l$=constant, the law of length is verified *i.e.*,

$$n \propto \frac{1}{l}.$$

Precautions :

(Same as in Expt. 36).

Sources of Error :

(Same as in Expt. 36).

ORAL QUESTIONS

(Same as in Expt. 36.)

Experiment 38.

To study the relation between length and diameter for same frequency and tension of a stretched string using a sonometer.

Apparatus :

A sonometer, 3 different wires, a hanger, half-kilogram weights, two wedges, wooden blocks, a tuning fork of known frequency, a pair of scissors, sensitive balance, a weight box, a metre rod, a rubber hammer and a screw gauge.

Theory :

(Same as in Expt. 36).

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

where m = mass per unit length of the wire.

$$= 1 \,.\, \pi \frac{D^2}{4} \,.\, \rho \quad \text{where } D = \text{diameter of the wire}$$

and ρ = its density.

$$= \frac{\pi D^2}{4} \,.\, \rho$$

$$\therefore \qquad n = \frac{1}{2l}\sqrt{\frac{T.4}{\pi D^2.\rho}}$$

$$\text{or } n = \frac{1}{lD}\sqrt{\frac{T}{\pi \rho}}$$

Since n, T and ρ are constant so

$$lD = \text{constant}$$

$$\text{or} \qquad l \propto \frac{1}{D}.$$

Procedure :

1. Stretch the wire with a load of 4 kg including the hanger and find the length of the wire which vibrates in resonance with a given tuning fork **as explained in Experiment 36.**

2. Find the diameter of this wire **as explained as Expt. 3.**

3. Repeat with the same load and the same tuning fork for three *different* wires.

Note : *The wires may be of different materials and different diameters.*

Observations and Calculations :

Load including the hanger =............*kg*

Frequency of the tuning fork $=n=$......Hz

Wire No.	*Length of the wire giving note of frequency n (in cm)*			*Corrected Diameter of the wire (D) in cm*	*l×D*
	Increasing	*Decreasing*	*Mean (l)*		

Diameter of the wire :

Least count of the screw gauge=............cm

Zero error=............

Zero correction=............cm

Verification :

$$\text{Since } l \times D = \text{constant so } l \propto \frac{1}{D}$$

Result :

For the same frequency (n) and tension (T) of a stretched string of a sonometer, the length (l) of the string is inversely proportional to its diameter (D).

Precautions :

(Same as in Expt. 36).

Sources of Error :

(Same as in Expt. 36).

ORAL QUESTIONS

(Same as in Expt. 36).

Experiment 39 :

To determine the frequency of A.C. mains using a sonometer. Taking the frequency of A.C. mains as 50 Hz, calculate the percentage error.

Apparatus :

A vertical pattern sonometer, (or Horizontal Sonometer), a solenoid with a soft iron core, a pan (or a hanger), half kg-weights, chemical balance and weight box.

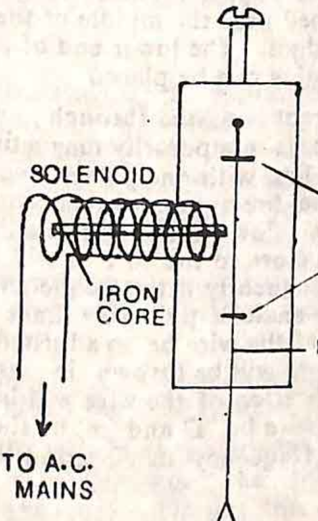


Fig. 39.1 Vertical sonometer for frequency of A.C. mai

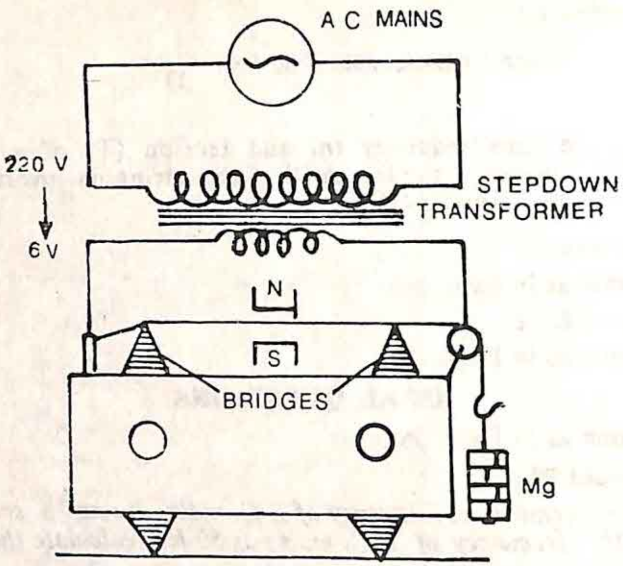


Fig. 39.2. Horizontal sonometer for frequency of A.C. mains

Theory :

The apparatus consists of a vertical pattern sonometer (Fig. 39.1) on which is stretched an *iron* wire. A solenoid having a large number of turns of insulated copper wire and carrying a soft iron core along its axis is clamped near the middle of the segment of the wire between the two bridges. The lower end of the wire carries a pan on which suitable weights can be placed.

If an alternating current is passed through a solenoid having a soft iron core, the core is temporarily magnetised twice during each cycle of alternation—first with one polarity when the oscillation of the current is in one direction, and then with the opposite polarity when the current flows in the opposite direction. When the sonometer wire is held close to the core, it will be pulled twice during each cycle and consequently if the frequency of the alternating current be '*n*', the wire shall be pulled 2*n* times per second. If the length and tension of the wire be so adjusted that its natural frequency is also 2*n*, the wire will be thrown in *resonant* vibration and the amplitude of vibration of the wire will be maximum. If the tension applied to the wire be 'T' and '*m*' be the mass per unit length of the wire, the frequency of vibration 'N' of the wire is given by

$$N=\frac{1}{2l}\sqrt{\frac{T}{m}}$$ where l is the resonant length of the wire.

The frequency (n) of the A.C. mains will be equal to $\frac{N}{2}$

Hence, $n = N/2$

or
$$\boxed{n=\frac{1}{4l}\sqrt{\frac{T}{m}}}$$

Note. Another variation of the apparatus is the *horizontal pattern sonometer* (Fig. 39.2) on which is stretched a brass wire. The alternating voltage is stepped down to say 6 volts by means of a step-down transformer and then is connected to the wire as shown. The wire passes between the pole pieces N and S of a permanent horse shoe magnet. The wire therefore experiences an alternating force due to the field of the magnet on the current in the wire, and for a particular length of the wire between the bridges it is thrown into resonance as is evidenced by a large amplitude. This condition is achieved when the frequency of the alternating current passing through the wire is equal to its mechanical frequency of vibration ; which is given by the formula

$$\boxed{n=\frac{1}{2l}\sqrt{\frac{T}{m}}}$$

The vertical pattern is preferable to the horizontal one, since the friction at the pulley is completely eliminated.

Procedure :

1. Before starting the actual experiment, have an idea of the breaking stress for the material of the wire from the Table of Physical constants. From this calculate the breaking tension (=breaking stress × area of cross-section of the wire) for your wire. *During subsequent experiment the weight in the pan should not exceed half the breaking tension.*

2 Suspend the sonometer from the nail on the wall and see that the pan provided below stays clear of the wall. Put a suitable load on the pan.

3. Switch on the current and adjust the core of the solenoid near the middle of the wire between the bridges.

4. With the help of the bridges adjust the length of the wire till it begins to vibrate under the influence of the magnetic field provided by the core. During this adjustment the core should always be placed near about the middle of the vibrating wire.

Now by a slight delicate adjustment attain a position when the wire is thrown in violent resonant vibration and the amplitude is maximum.

5. Switch off the current and measure the length of the vibrating wire by holding a scale on the bridges and avoiding the error due to parallax. Record the tension *which should include the mass of the pan or the hanger.*

6. Vary the tension in suitable steps and obtain the corresponding lengths of the vibrating wire.

7. Now weigh in a chemical balance a known length (say 100 cm.) of the sonometer wire and thus calculate mass per unit length of wire (m).

8. Calculate the frequency of the A.C. mains as indicated below.

Observations :

Mass of 100 cm of wire=...g

∴ Mass per unit length=m=...g/cm

S. No.	*Tension applied to the wire (including mass of pan)*	*Length of the Resonating wire (l)*	*Mean Tension (T) in dynes*	l^2	*Mean (l^2)*
1.	.........dynes	.........cm	.........dynes	....	......cm^2
2.	....	....		....	
3.	....	....		....	
4.	....	....		....	

Calculations :

Substituting the means values of T and l^2 in the formula

$$n=\frac{1}{4l}\sqrt{\frac{T}{m}}$$

we have $$n^2=\frac{1}{m}\left(\frac{T}{16l^2}\right)$$

$$=..........$$

Hence n=...........Hz.

Result :

The frequency of the A.C. Means by experiment=.........Hz

% Error=..............

Precautions and Sources of Error :

1. The sonometer wire should be uniform and free from kinks.

2. For bringing the wire in resonant vibration, start with a small length of the wire and increase the length in small steps. The solenoid should be so placed that the iron core is situated close to the middle of the wire.

3. While finding out the tension of the wire, do not forget to add the mass of the pan or of the hanger. If a sonometer employs a *spring balance* note down the *zero error*, if any.

4. While increasing the tension of the wire, be careful that the wire is not stretched beyond elastic limit. For this purpose, before starting the experiment have an idea of the magnitude of the breaking load of the given wire from the Table of Physical constants.

5. In the derivation of the formula $V=\sqrt{\frac{T}{m}}$ it has been assumed that the wire is perfectly flexible. Hence due to the rigidity of the experimental wire an *error* shall creep in the result.

6. If the wire is not uniform or if its composition is variable then also the result will be erroneous.

7. The tension on the two sides of the bridges may not be the same.

8. If the horizontal pattern of the sonometer is employed, there will be an additional source of error. There may be friction at the pulley, hence the value of tension is less than that actually applied. This consequently affects the value of the frequency.

ORAL QUESTIONS

[For Questions on **sonometer** see Expt. No. 36.]

Q. 1. *What do you mean by A.C. mains ?*

Ans. "*Main*" stands for the alternator or the machine which is supplying current to the town. 'A C.' means that the current supplied by the generator is not *unidirectional* but is *alternate* in nature *i e.*, the current is varying at every instant and after a fixed interval of time it even gets *reversed* in direction (Fig. 39·3).

Q. 2. *What is meant by the frequency of A.C. ?*

Ans. If, at any particular moment, an alternating current has a certain value and is just going to commence a certain set of variations, then the time which elapses between this instant and the moment when the current has the same value and is going to commence an identical set of variations is called the *period* and the number of periods in one second is called the *frequency*.

An alternating current is represented by

$$I = I_0 \sin \theta$$

Fig. 39.3 represents the type of variations, the current is undergoing. Points A, B, C...are in the *same phase*. The time taken for the wave to travel from A to B is one period and the number of such variations in one second will give the frequency of the A.C.

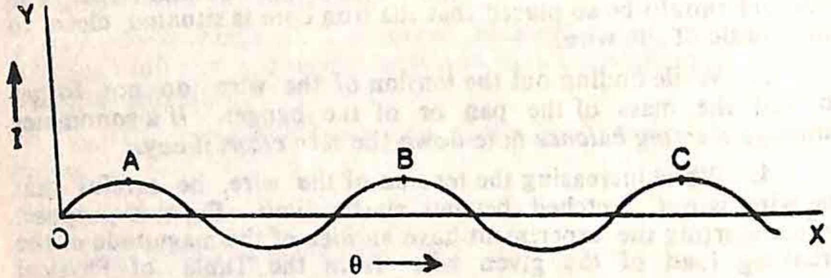


Fig. 39.3

Q. 3. *What is the frequency of the mains of Delhi supply ?*

Ans. It is 50 cycles per second or Hz.

Q. 4. *What is meant by the statement "50 cycles/sec" ?*

Ans. It means that in one second the current flows 50 times in one direction and then 50 times in the reverse direction.

Q. 5. *Does the current become zero also in this process ?*

Ans. Yes. It does.

Q. 6. *How many times in one second ?*

Ans. 100 times in one second.

Q. 7. *What types of waves are being produced in the wire of the sonometer in this experiment ?*

Ans. Transverse stationary waves.

Q. 8. *What is the distance between a node and an antinode ?*

Ans. It is equal to $\frac{\lambda}{4}$ *i.e.*, equal to one-fourth of the wave length.

Q. 9. *How do you test resonance in this experiment ?*

Ans. We test the resonance by placing the iron core of the solenoid near the middle of the wire between the bridges. Resonance takes place when maximum amplitude is obtained.

Q. 10. *What is the construction of the solenoid ?*

Ans. It consists of a large number of closely wound turns of insulated copper wire, along the axis of which is placed a soft iron core.

Q. 11. *During the adjustment for the resonant length, we change the position of the solenoid. Why do we do so ?*

Ans. Each time we put the solenoid in such a way that the core is near the middle of the wire between the bridges. When the wire vibrates, an antinode is situated at its middle point, and at this point, we place the core so that it applies maximum pull here.

Q. 12. *If an antinode is formed at the middle point of the wire, then there must be nodes somewhere. Where are they ?*

Ans. They are situated at places where the wire touches the bridges.

Q. 13. *What is this distance between the nodes equal to ?*

Ans. It is equal to $\frac{\lambda}{2}$ *i.e.*, equal to half the wavelength.

Q. 14 *Then the frequency of the vibrating wire should be given by the ordinary sonometer formula. Is it not ?*

Ans. Yes. It is.

Q. 15. *Then the frequency of the mains will also be obtained by the same formula, since the two are in resonance. Is it now so ?*

Ans. No. It is not exactly so. To get the frequency of the A.C. mains we value the frequency of the wire.

Q. 16. *Why is it done so ?*

Ans. When alternating current is passed through the solenoid, the iron core situated within it is temporarily magnetised twice during each cycle of alternation—first with one polarity when the oscillation of the current is in one direction, and then with the opposite polarity when the current flows in the opposite direction during the next half cycle. Now the sonometer wire is held quite close to the core, consequently it is pulled *twice* during each cycle of the current. Obviously, if the frequency of A.C. be '*n*', the wire will be pulled '2*n*' times per second. If the length and tension of the wire is so adjusted that its natural frequency is also 2*n*, the wire is thrown in resonant vibration and the amplitude of vibration of the wire becomes maximum. It is for this reason that the frequency of the sonometer wire is first calculated with the usual sonometer formula and then to evaluate the frequency of the A.C. mains, the result is halved.

Q. 17. *You are using an iron wire here in this experiment. Cannot you use a brass wire ?*

Ans. Yes. A brass wire can also be used, but in that case the wire should carry a D.C. through it. Now the wire shall experience an attractive force due to the core of the solenoid. Alternatively, with a brass wire the arrangement of the sonometer can be modified.

Q. 18. *How?*

Ans. In this case the A.C. voltage is transformed from 220 volts to say, 6 volts by means of a step-down transformer and this voltage is fed to the ends of the brass wire of the sonometer (Fig 39 2). The wire passes in between the pole pieces of a permanent horse-shoe magnet. Due to the interaction of the magnetic field of the magnet on the current in the wire, the latter experiences a force tending it to move. This force changes direction during each half cycle of the current, consequently the wire begins to oscillate.

Q. 19. *In this case when is the wire thrown into resonant vibration?*

Ans. Resonance occurs when the mechanical frequency of the transverse vibrations of the stretched wire equals that of the A.C. Thus the frequency of the A.C. mains is given by the formula

$$n=\frac{1}{2l}\sqrt{\frac{T}{m}}.$$

Q. 20. *How can you determine the direction of the force experienced by a wire carrying current and placed in a magnetic field?*

Ans. We can determine the direction of the force by using Fleming's Left Hand Rule which is as follows :

Stretch the first finger, the middle finger and the thumb of your left hand in such a way that they are mutually at right angles to each other. If the first finger represents the direction of the magnetic field, the middle finger gives the direction of the current in the wire, then the thumb will represent the direction of force or the direction of motion of the wire.

Experiment 40 :

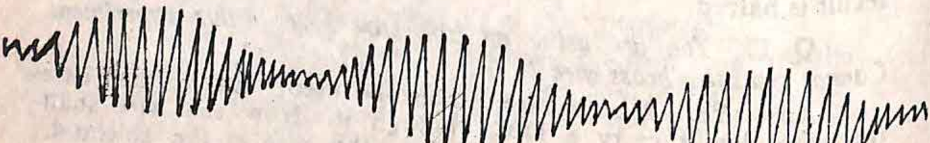
To find the difference in the frequencies of two tuning forks by counting their beats and identifying one of higher frequency.

Apparatus :

Two tall tuning forks of the same frequency mounted on resonating wooden boxes, rubber hammer, plasticine or wax or a movable mass (a small load with adhesive tape), stop watch or stop-clock.

Theory :

Beats :



VARIATIONS OF AMPLITUDE IN BEATS

Fig. 40.1

When two pure notes of nearly the same pitch are sounded together, periodic variations in the intensity of the sound are heard. These alterations of sound and comparative silence are termed **beats.** They can be plainly recognised when two tuning forks of nearly the same frequency are set in vibrations together. If the forks have frequencies N_1 and N_2 respectively, the *number of beats per second* is the difference between these frequencies, $n=N_1-N_2$. Here N_1 is supposed greater than N_2.

This result can be explained by the principle of interference. The velocity of propagation is the same for the two notes, but the wavelengths differ slightly. Where the waves agree in phase they will strengthen each other, but where they are opposed they will neutralise one another (Fig. 40.1). Let us take a starting point an instant when waves from the two sources reach the ear in the same phase. At the end of one second, the higher note has made N_1 complete vibrations, the lower note only N_2. that is, the higher note has made (N_1-N_2) more vibrations than the lower. During the second, one system of waves has been falling behind the other, and the loss amounts to (N_1-N_2) wave lengths. Hence there must have been (N_1-N_2) occasions in the course of the second when the two systems agreed in phase, and (N_1-N_2) occasions when the phases were opposed so that there was comparative silence. In other words, the number of beats in one second $n=N_1-N_2$.

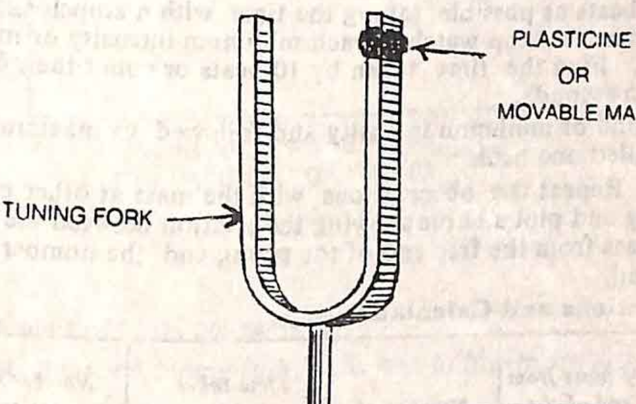


Fig. 40.2

If the two notes are nearly in unison, the beats are very slow and it is difficult to distinguish them. On the other hand, if the number of beats is more than four per second, it is difficult to count them. When the beats become so rapid that they cannot be separately perceived, a *discord* or *dissonance* is produced.

Procedure :

1. Take two tall tuning forks of the same frequency mounted on resonant wooden boxes. Make their frequencies slightly different by loading one by plasticine or wax or by tightly attaching a small load by adhesive tape so that it can be clamped at any part of the prong (Fig. 40.2).

Note : Both the tuning forks must be of rather good quality and must give audible sound for about 8 to 10 seconds inspite of dissipation of energy in the resonating box.

2. Strike the tuning forks with a rubber hammer in *quick succession, with roughly equal force* ; carefully listen to the combined sound produced by the two tuning forks. Gradual increase and decrease in the intensity of sound will be heard. It is due to the beats produced by the superposition of waves of slightly different frequencies.

Count the number of beats in a measured interval of time. The number of beats per second should be determind by counting as many beats as possible, taking the time with a stopclock or stop-watch. Start the stop-watch at each minimum intensity or maximum intensity. Find the time taken by 10 beats or count the number of beats in 5 seconds.

Sound of minimum intensity and followed by maximum intensity is called one beat.

3. Repeat the observations with the mass at other points on the prong and plot a curve showing the relation between the distance of the mass from the free end of the prong and the number of beats per second.

Observations and Calculations :

Distance of mass from the free end of the prong (cm)	*Number of beats* (*a*)	*Time taken* (*t*) *in sec*	*Number of beats per second* $=n=\frac{a}{t}$
1.			
2.			
3.			

If N_1 is the frequency of the unloaded tuning fork and N_2 is the frequency of the loaded tuning fork and since by loading, the frequency gets decreased, so the tuning fork of frequency N_1 is the one of higher frequency.

Number of beats per second

$$= n = N_1 - N_2 = \frac{a}{t} = \ldots\ldots$$

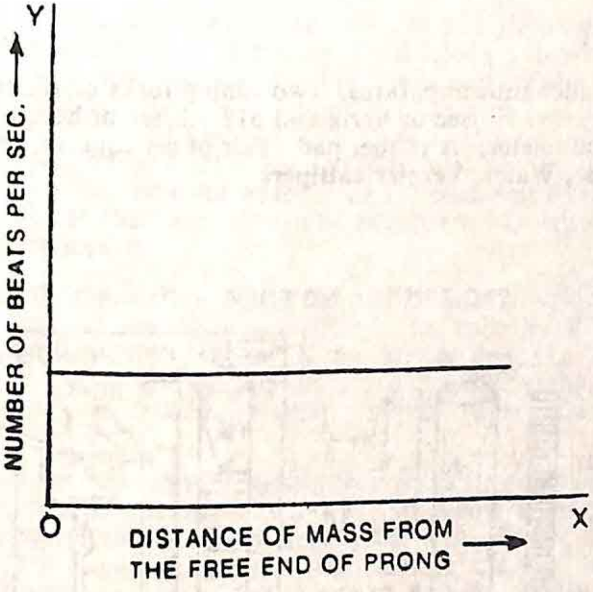


Fig. 40.3

Result :

(*a*) Number of beats per sec=.........

(*b*) The unloaded tuning fork is the one of higher frequency.

Precautions :

1. Two tuning forks should be of *identical* frequency and should be of good quality giving audible sound for about 8 to 10 seconds.

2. There should be pin-drop silence in the laboratory.

3. Strike the tuning forks in quick succession with *equal* force.

4. Place the two tuning forks close to each other.

5. Start the stop watch either at minimum intensity or at maximum intensity.

ORAL QUESTIONS

(Same as in Expt. 36)

Experiment 41 :

To determine the wavelength of sound by taking first and second resonance positions and explain the difference, if any and hence determine velocity of sound at room temperature and the end correction. Also find the relation between frequency and Wavelength.

Apparatus :

Resonance tube apparatus, Two tuning forks of different frequencies (say 480 vib/sec or hertz and 512 vib/sec or hertz) or more, Celsius thermometer, A rubber pad, Pair of set squares, A beaker, A plumb line, Water, Vernier callipers

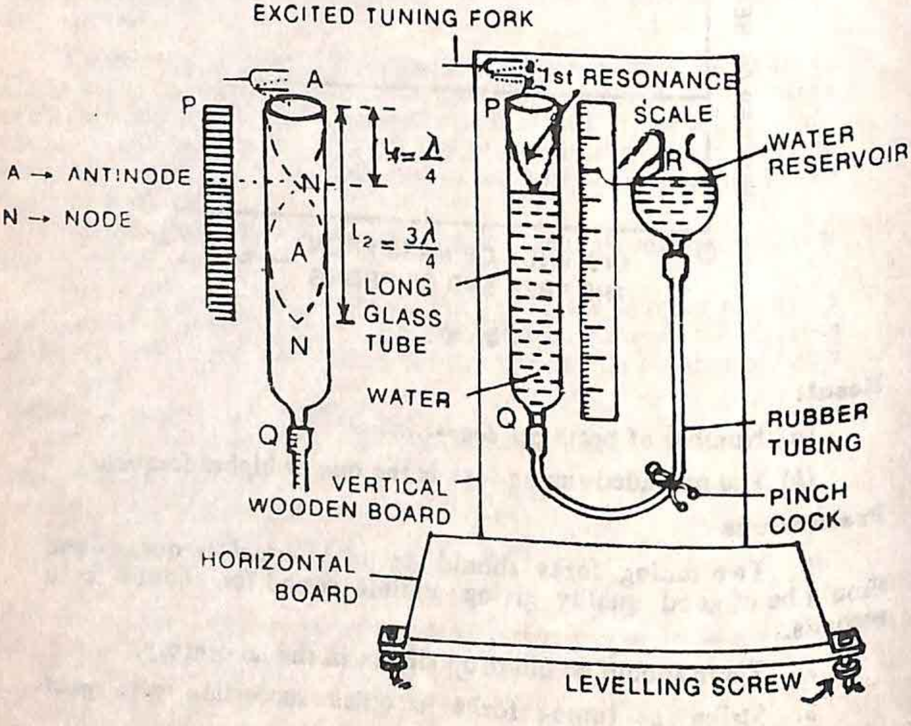


Fig. 41.1. Resonance Tube Apparatus

Theory :

Resonance is the phenomenon of vibratory motion produced in a body by the influence of the vibrations of another body when their frequencies are exactly the same.

An air column can easily be set into vibrations with the help of a tuning fork. The frequency of the air column can be adjusted equal to that of the tuning fork by varying its length. This can be easily achieved with the help of an apparatus called *resonance tube* (Fig. 41.1).

The resonance tube apparatus consists of a glass tube PQ about 1 metre long and 5 cm in diameter fixed along a metre scale on a vertical wooden board or stand. The lower end of the tube is connected by a rubber tube to a reservior (R) which can slide up and down along the scale and can be clamped in any desired position. A part of the tube and reservoir are filled with water. By means of a pinch-cock, the flow of water from the reservoir to the tube and from the tube to the reservoir can be regulated. The base is provided with levelling screws.

Let a tuning fork of frequency '*n*' be struck against a rubber pad and held at the open end (P) of the tube as shown in Fig. 41.1 Adjust the level of water in the tube so that the air column begins to vibrate in resonance with the tuning fork and a loud sound is heard. Let us understand what happens when the *air column resonates.*

When the prong of the tuning fork starts its downward journey from mean position, it sends down in the tube a wave of compression that reaches the level of water at B and is reflected at a denser medium as a wave of compression. If the wave of compression reaches P, when the prong of the tuning fork reaches its mean position the sound becomes very loud and resonance occurs. It is because the tuning fork after reaching its mean position starts its upward journey and sends down in the tube a wave of rarefaction. The previous wave of compression is reflected at P as a wave of rarefaction since the reflection takes place at rarer medium because the air inside the tube being in a state of compression is comparatively denser than the outside air. The two waves of rarefactions thus, superimpose on each other producing *stationary waves* and the amplitude of the air particles increases, thereby resulting in the increase of intensity in loudness. The open end of the tube always acts as an *antinode*—a point where amplitude is the greatest and the changes of pressure and density are zero and closed end as a *node*—a point where the amplitude is zero but the changes of pressure and density are maximum.

It is clear from above that during the time the prong completes *half a vibration*, the wave travels a distance equal to *twice* the length of the air column. Hence the wave travels four times the length of the air column during the time the prong completes one vibration. Since the distance that a wave travels in one period is equal to one wavelength, therefore, the wavelength—'λ' of the note is four times the length l_1 of the air column *i.e.*,

$$\boxed{\lambda=4l_1}$$

Similarly it may be argued that for the second resonance position during the time the prong makes three half vibrations the wave travels twice the length of the air column as shown in Fig. 41.1. If 'l_2' denotes the length, then $\frac{3\lambda}{2}=2l_2$

or $$\boxed{\frac{3\lambda}{4}=l_2}$$

or $$\boxed{\lambda=\frac{4l_2}{3}}=4l_1$$

$$\therefore \quad l_2=3l_1.$$

End correction and velocity of sound at room temperature.

It has been, however, shown by Lord Rayleigh that the antinode is not just at the open end (P) of the tube but slightly above it; so a small correction is to be applied in the observed length of the air column. This correction is known as *End correction* and is found to be 0·3 D, where D is the *internal diameter* of the resonant tube. Thus for the first resonance ;

$$\lambda=4[l_1+0{\cdot}3\ \mathrm{D}]$$

and for second resonance position

$$\lambda=\frac{4}{3}\Big[l_2+0{\cdot}3\ \mathrm{D}\Big]$$

The end correction 'x' can be determined experimentally as follows :

For 1st resonance $l_1+x=\frac{\lambda}{4}$ ...(*i*)

and for second resonance $l_2+x=\frac{3\lambda}{4}$ ...(*ii*)

Multiply (*i*) by 3 and subtract it from (*ii*), we have

$$3(l_1+x)=l_2+x$$

or $$\boxed{x=\frac{l_2-3l_1}{2}}$$

Subtracting (*i*) from (*ii*), we have

$$l_2-l_1=\lambda/2$$

$\therefore$ $$\lambda=2(l_2-l_1)$$

Hence $$V=n\lambda$$

or $$\boxed{V=2n\,(l_2-l_1)}$$

where V is the velocity of sound at room temperature.

Procedure :

1. Set the resonance tube vertical with the help of plumb line and levelling screws so that the vertical board is equidistant from the thread of the plumb line at every point.

2. Pour water in the reservoir (R) and the tube (PQ). Test the pinch-cock *i.e.*, the water level in the tube does not change when the pinch-cock is closed and note down the room temperature in °C.

3. Strike gently a tuning fork of known frequency say 512 hertz on the pad. Hold it above the open end of the tube so that its prongs are horizontal and vibrate in a vertical plane as in Fig. 41.1.

Now adjust the level of water in the resonance tube by raising and lowering the reservoir till you hear a sound of maximum intensity. The air column is said to vibrate due to '*resonance*'. Note the position of the water level with the help of a set square on the scale. Note the length of the resonance air column. This position corresponds to the first resonance position.

Lower the water level in the tube further by few cms and then close the pinch-cock and raise the reservoir a few centimetres. Now gently open the pinch-cock so as to allow the water level to rise slowly. Replace the vibrating tuning fork and note the position B of the water level at which the intensity of sound is maximum. Note the length of air column again. The mean of these two readings gives the length l_1. Confirm the resonance position by taking four readings, two when the water level is *falling* and the other two when the water level is *rising*.

4. Lower down the reservoir to the bottom so that the length of air column is approximately, increased three times and proceed in the same way to find the second resonance position. Take the mean of two readings for the second resonance with the water level falling and rising to get 'l_2'. Repeat these observations thrice.

5. Repeat the experiment with another tuning fork of known frequency (say 480 Hz) and find the mean of two values of the velocity of sound in air at room temperature.

6. Measure the internal diameter of the tube with the help of vernier callipers in different positions.

7. Note the room temperature with the help of the thermometer.

Observations and Calculations :

Room temperature at the start of the experiment

$=t_1=$............°C

Room temperature at the end of the experiment

$=t_2=$............°C

Mean room temperature$=t$°C$=\dfrac{t_1+t_2}{2}=$............°C

Vernier constant of vernier callipers=............cm

Zero correction of the vernier callipers=............cm

Observed internal diameter of the tube

(1)=.........cm

(2)=.........cm

(3)=.........cm

Mean corrected internal diameter (D)

of the resonance tube=......cm.

Position of the upper end of the tube=......cm

Result :

(*i*) An air column can be easily set into resonant vibrations with the help of a tuning fork.

(*ii*) Wave length of the sound wave=......cm=......metre

(*iii*) Velocity of sound in air at room temperature

t°C=......cm/sec=......m/sec

(*iv*) End correction=......cm

Precautions :

1. The resonance tube should be vertical.

2. The tuning fork should be gently struck against a soft rubber pad. It should *never be banged* as this process will in time cause a slight change in frequency.

3. The prongs of the tuning fork should not touch the edge of the tube and their ends should remain in the centre of the tube.

4. The resonance should be obtained for water level rising as well as falling in the tube.

5. The tuning fork should not be roughly handled.

6. Reading of the lower meniscus of the water level should be noted with set squares.

Sources of Error :

(1) The presenee of moisture in the tube will raise the velocity of sound.

(2) Position of the resonance especially second resonance cannot be accurately determined.

Note : Before starting the experiment the velocity of sound in cm/s at room temperature is calculated by $V_t=33200+61t$ where t is the room temperature and hence the approximate position of the 1st resonance is determined by $l_1=\frac{V_t}{4n}$ cm. Now the experiment should be started by keeping the water level at this position.

ORAL QUESTIONS

Q. 1. *Nrme the various types of vibrations ?*

Ans. There are three types of vibrations.

(*a*) Free (*b*) Force and (*c*) Resonant vibrations.

Q. 2. *What are free vibrations ?*

Ans. When a body vibrates with its natural frequency, without being disturbed by any other external force, it is said to have free vibrations.

Q. 3. *What are forced vibrations ?*

Ans. A body is said to execute forced vibrations if it is made to vibrate with a frequency other than its natural frequency by the application of some external force.

Q. 4. *What are resonant (or sympathetic vibrations) ? What is resonance ?*

Ans. A body is said to execute resonant (or sympathetic) vibrations if it is made to vibrate with its natural frequency by the application of some external force having the same frequency as that of the body. This phenomenon of producing vibratory motion in a body by the influence of some external force (or a second vibrating body) having the same natural frequency as the first is called *resonance*.

Q. 5. *What is resonance column ?*

Ans. It is the air column between the surface of water and the tuning fork.

Q. 6. *What is the function of water in the resonance tube apparatus ?*

Ans. Water acts as a denser medium and reflects the longitudinal waves produced in air due to vibration of prongs of tuning fork.

Q. 7. *Can we use any other liquid ? Why water ?*

Ans. Yes ; any liquid can be used. Water is used because it is cheapest of all.

Q. 8. *What types of waves are produced in the tube ?*

Ans. Stationary Waves (or Standing Waves).

Q. 9. *Is there node or antinode (i) at the upper end of the tube, (ii) at the surface of water ?*

Ans. There is an antinode at the upper end of the tube and a node at the surface of water.

Q. 10. *When does resonance occur ?*

Ans. When the frequency of vibrating air column becomes equal to that of the tuning fork and thus maximum sound is produced due to resonance.

Q. 11. *What is the use of pinch-cock ?*

Ans. It is used for adjusting the level of water in the resonance tube.

Q. 12. *What is End correction ?*

Ans. The reflection of longitudinal waves at the upper end of the resonance tube takes place at a position which is a little higher than the open end. This distance above the upper end is called End-correction. It is numerically equal to 0·3 times the internal diameter of the tube. The error is called End error.

Q. 13. *Can we take resonance tube of square cross-section ?*

Ans. Yes : it can have any shape.

Q. 14. *Why do we use a long tube ?*

Ans. So that we may get two resonance positions and thus end correction may be eliminated.

Q. 15. *What is the effect of temperature, density and pressure on the velocity of sound ?*

Ans. The velocity of sound is directly proportional to the square root of absolute temperature ; inversely proportional to the square root of density while pressure *has no effect*.

Q. 16. *In which medium is the velocity of sound greatest—iron ; water or air ?*

Ans. It is greatest in iron, less in water and least in air.

Q. 17. *Are the waves in the resonance column and sonometer of the same nature ?*

Ans. No ; the waves in resonance column are longitudinal while in sonometer there are transverse waves.

Q. 18. *Are the waves in resonance column and sonometer progressive or stationary ?*

Ans. The waves in both the cases are stationary.

Q. 19. *How should the tuning fork be placed over the resonance tube.*

Ans. The tuning fork be placed just above the open end with its length perpendicular to the length of the air column.

Q. 20. *Why do you take two positions of resonance in the tube ?*

Ans. To eliminate End-correction.

Q. 21. *Why is the first resonance produced at a distance of* $\frac{\lambda}{4}$ *and not at* $\frac{\lambda}{2}$ *?*

Ans. Under the condition of first resonance, the resonating length of the tube is travelled twice by the compression and twice by the rarefaction corresponding to one wavelength (λ). Therefore the resonating length of the tube is one quarter $\left(\frac{\lambda}{4}\right)$ of a wave length. Moreover this resonating length is the shortest length of the resonance column of air, with an antinode at the open end and a node at the water surface.

Q. 22. *Why do you get the second position at approx. three times the first resonant length ?*

Ans. In the second position at resonance, the fork completes three vibrations in the time the sound travels four times (two times compression and two times rarefaction) the length of the tube. Therefore, in this case the length of the air column is equal to three quarters of the wavelength *i.e.*, approximately three times the first resonant length.

Q. 23. *Why is the second resonance feebler than the first ?*

Ans. At the second resonance, the vibration in the air column corresponds to the first overtone which is very weak.

Experiment 42 :

To compare the frequencies of two tuning forks using the resonance tube and find the end correction.

Apparatus :

(Same as in Expt. 41).

Theory :

(Same as in Expt. 41).

Procedure :

1. Proceed as in **Experiment 41** and find the length of first resonance air column (l_1) and the length of second resonance air column (l_2) with tuning fork of frequency n_1.

2. Repeat the experiment with another tuning fork of frequency n_2 and find the length of first resonance air column (l'_1) and length of second resonance air column (l'_2).

3, Enter the observations & calculations in the table given on page 228.

Verification :

(*i*) $\frac{n_1}{n_2}$=........... (The values of n_1 and n_2 are written on the tuning forks.)

(*ii*) **End Correction:**

Vernier constant of vernier callipers=......cm

Zero correction of vernier callipers=......cm

Observed internal diameter of the tube

(1) ....cm (2) ....cm (3) ....cm

Mean corrected internal diameter (D) of the resonance tube=......cm

End correction=x=0·3D=......cm

Result :

(*i*) $\frac{n_1}{n_2}=\frac{l'_2-l'_1}{l_2-l_1}$=.........

(*ii*) End correction=.........cm

Precautions :

(Same as in Expt. 41).

Sources of Error :

(Same as in Expt. 42).

ORAL QUESTIONS

(Same as in Expt. 41).

Observations and Calculations :

Tuning fork of frequency	*S. No.*	*FIRST RESONANCE*			*Mean* (l_1) *in cm*	*SECOND RESONANCE*			*Mean* (l_2) *in cm*	$\frac{n_1}{n_2}=\frac{l'_2-l'_1}{l_2-l_1}$	*End correction* (x) *in cm*
		Position of water level				*Position of water level*					
		Falling	*Rising*	*Mean*		*Falling*	*Rising*	*Mean*			
n_1	1										$x=\frac{l_2-3l_1}{2}$
	2				$l_1=$ cm				$l_2=$ cm		
	3										
n_2	1										$x=\frac{l'_2-3l'_1}{2}$
	2				$l'_1=$ cm				$l'_2=$ cm		
	3										
										Mean=	

TABLES OF PHYSICAL CONSTANTS

SOME USEFUL CONSTANTS AND TABLES

TABLE 1

Densities of Some Common Substances in kg. m^{-3}

Asbestos	=2400	Graphite	=2300
Alcohol (ethyl)	=790	Glass (Crown)	=2500
Alcohol (methyl)	=810	Glass (Flint)	=2900 to 4500
Alum	=2500	Glass (common)	=2400 to 2800
Aluminium	=2700	Ice	=916
Coal	=1200 to 1500	Platinum	=21850
Wood	=600 to 800	Rubber (India)	=910 to 950
Iron (wrought)	=7850	Sand	=2300 to 2600
Iron (cast)	=7600	Marble	=2500 to 2800
Brass	=8400 to 8700	Mercury	=13600
Copper	=8900	Lead	=11370
Cork	=180 to 260	Copper Sulphate Crystal	=2100
Coin (old)	=10300	Silver	=10500
Sugar	=1590	Kerosene Oil	=800
Steel	=7700	Turpentine Oil	=870
Stainless steel	=7800	Olive Oil	=900
Glycerine	=1260	Wax	=900
Spirit	=830	Milk	=1030
Petrol	=800	Zinc	=7100
Sod. Chloride	=2150	Manganin	=8500
Gold	=19300		

TABLE 2

Acceleration due to gravity at different places in India along with their Latitude, Longitude and Elevation

Place	g (m/s^2)	*Lat.* (*N*)	*Long.* (*E*)	*Elevation* (*m*)
Agra	9·7905	27°12′	78°02′	158
Aligarh	9·7908	27°54′	78°05′	187
Allahabad	9·7894	25°27′	81°51′	94
Banaras	9·7893	25°20′	83°00′	81
Bombay	9·7863	18°54′	72°49′	10
Calcutta	9·7880	22°35′	88°20′	6

Delhi	9·7914	28°40′	77°14′	216
Equator	9·7805	00°00′	na	0
Jaipur	9·7900	26°55′	75°47′	433
Udaipur	9·7881	24°35′	73°44′	563
Srinagar	9·7909	34°05′	74°50′	159
Pole	9·8322	90°00′	na	0
Madras	9·7828	13°04′	80°15′	6
Trivandrum	9·7812	8°28′	76°58′	27
Tirupati	9·7822	13°38′	79°24′	169
Madurai	9·7810	9°55′	78°07′	133
Bangalore	9·7803	12°57′	77°37′	915
Gauhati	9·7899	26°12′	91°45′	52
Bhubaneswar	9·7866	20°28′	85°54′	23

TABLE 3

Variation of atmospheric Temperature and Pressure with Altitude

(At sea level pressure=Standard Atmosphere and Temperature=15°C assum.)

Altitude (metres)	*Pressure (millibars)*	*Temperature* (°C)
0	1013·25	15·0
250	983·58	13·4
500	854·61	11·8
750	926·34	10·1
1000	898·75	8·5
1500	825·56	5·2
2000	794·95	2·0
2500	746·82	−1·2
3000	701·08	−4·5
3500	657·64	−7·8
4000	616·40	−11·0
4500	577·28	−14·2
5000	540·20	−17·5
6000	471·81	−29·0
7000	410·61	−30·5
8000	356·00	−37·0
9000	307·42	−43·5
10000	264·36	−50·0

TABLE 4
Surface Tension of Liquids

Substance	*In contact with*	*Temp (°C)*	*Surface Tension* (10^{-3} Nm^{-1})
Water	Air	10	74·22
	Air	20	72·55
	Air	30	71·18
	Air	40	69·56
	Air	50	67·91
Acetic acid	Vapour	10	28·8
	Vapour	20	27·8
	Vapour	50	24·8
Ethyl Alcohol	Air	0	24·05
	Vapour	10	23·61
	Vapour	20	22·75
	Vapour	30	21·89
Glycerol	Air	20	63·4
	Air	90	58·6
Methyl Alcohol	Air	0	24·49
	Air	20	22·61
	Vapour	50	20·14
Mercury	Vapour	20	470
	Vapour	100	456
Oleic acid	Air	20	32·5
Kerosene	Air	20	24
Turpentine	Air	20	27

TABLE 5
Elastic Properties of Solids

Substance	*Young's Modulus* (10^{10} Nm^{-2})	*Modulus of rigidity* (10^{10} Nm^{-2})	*Bulk Modulus* (10^{10} Nm^{-2})	*Breaking stress* (kg/mm^2)
Aluminium	7·03 to 7·05	2·61	7·55	20 to 25
Brass (70/30)	10·06	3·73	11·18	30 to 50
Copper	12·98	9·83	13·78	40 to 45
Gold	7·8	2·7	21·7	—
Iron (soft)	21·14	8·16	16·98	—
Silver	8·27	3·03	10·36	—
Steel (mild)	21·19	8·22	16·92	40 to 45
Silver	7·1 to 7·4			28
Rubber	0·05	0·00015	—	—
Wood (oak)	1·3	—	—	—
Wood (teak)	1·7	—	—	—
Glass	5·1—7·1	3·1	3·75	—
Quartz	5·4	3·4	—	—

TABLE 6

Velocity of Sound

(At 20°C unless otherwise stated)

Substance	*Velocity of longitudinal wave* (ms⁻¹)	*Substance*	*Velocity of longitudinal wave* (ms⁻¹)
Alcohol	1177	Hydrogen (0°C)	1284
*Aluminium	5240	*Iron	5170
Air (0°C)	331·45	Mercury	1451
*Brass	3130—3450	Nitrogen (0°)	334
*Copper (annealed)	3790	*Steel (tool)	5150
Carbon dioxide (0°C)	259	Water	1484
*Glass, Crown	4710—5300	Water vapour (100°C)	405
*Glass, flint	3490—4550	Oxygen (0°C)	316

TABLE 7

Standard Wire Gauge

Size (*S.W.G.*)	*Diameter* (mm)	*Size* (*S.W.G.*)	*Diameter* (mm)
1	7·62	16	1·63
2	7·01	17	1·42
3	6·40	18	1·22
4	5·89	19	1·02
5	5·38	20	0·914
6	4·88	21	0·813
7	4·47	22	0·711
8	4·06	23	0·610
9	3·66	24	0·559
10	3·25	25	0·508
11	2·95	26	0·457
12	2·64	27	0·417
13	2·34	28	0·376
14	2·03	29	0·345
15	1·83	30	0·315
		31	0·295
		32	0·274
		33	0·254
		34	0·234
		35	0·213
		36	0·193
		37	0·173
		38	0·152
		39	0·132
		40	0·122

*In case of solids velocities of longitudinal waves in thin rods are quoted.

TABLE 8

Coefficient of friction of common things

Substance	*Condition*	*Coefficient of dynamic friction*
Glass on glass	Clean and dry	0·18
Wood on glass	Clean and dry	0·2 to 0·3
Wood on wood	Clean and dry	0·25—0·5
Wood on steel	Clean and dry	0·20 to 0·25
Steel on steel	Clean and dry	0·17—0·23
Steel on steel	Greased	0·05
Stone on concrete	Dry	0·45
Car tyre on concrete	Moderate speed	0·40

TABLE 9

Coefficient of expansion

Solids	*Coefficient of linear expansion* $10^{-6}K^{-1}$	*Liquids*	*Coefficient of volume expansion* $10^{-4}K^{-1}$
Aluminium	24	Alcohol (ethyl)	11·2
Brass	18 to 19	Alcohol (methyl)	12·2
Copper	16·7	Benzene	12·4
Constantan	18	Ether (ethyl)	16·3
Glass (Pyrex)	3	Glycerine	5·3
Glass (soft soda)	8·5	Mercury	1·8
Iron (cast)	10·0	Water (15°C)	1·5
Iron (wrought)	12·0	Water (99°C)	7
Ice	51·0	Kerosene Oil	10·0
Steel	11·0	Turpentine	9·4
Lead	23·0	Olive Oil	7·2
Zinc	28·0		
Platinum	8·9		
Silver	19·2		

TABLE 10

Density of Water at Various Temperatures (in gm/cm³)

Temp °C	0	2	4	6	8	10	12	14	16	18
0	·99987	·99997	1·00000	·99997	·99988	·99973	·99953	·99927	·99897	·99862
20	·99822	·99780	·99732	·99681	·99626	·99567	·99505	·99440	·99371	·9930
40	·9922	·9915	·9907	·9898	·9890	·9881	·9872	·9862	·9853	·9843
60	·9832	·9822	·9811	·9801	·9787	·9778	·9767	·9755	·9743	·9731
80	·9718	·9706	·9693	·9680	·9667	·9653	·9640	·9626	·9612	·9598
100	·9584	—	—	—	—	—	—	—	—	—

TABLE—11

Boiling Points of Water at Various Barometric Heights

mm	0	1	2	3	4	5	6	7	8	9
700	97·709	748	788	827	866	906	945	984	98·023	062
710	98·102	141	180	219	258	296	335	374	413	451
720	98·490	529	567	606	644	683	721	759	798	816
730	98·874	912	950	989	99·027	065	102	140	178	216
740	99·254	292	329	367	405	442	480	517	554	592
750	99·629	666	704	741	778	815	852	889	926	963
760	100·00	037	074	110	147	184	220	257	294	330

TABLE 12

Vapour Pressure of Water at Various Temperatures

Temp. °C	0	1	2	3	4	5	6	7	8	9
0	4·579	4·924	5·290	6·681	6·097	6·541	7·011	7·511	8·042	8·606
10	9·205	9·840	10·513	11·226	11·980	12·779	13·624	14·517	15·460	16·456
20	17·51	18·62	19·79	21·02	22·32	23·69	25·13	26·65	28·25	29·94
30	31·71	33·57	33·53	37·59	39·75	42·02	44·40	49·90	49·51	52·26

Tem. °C	0	2	4	6	8	10	12	14	16	18
40	55·13	61·30	68·05	74·43	83·59	92·30	101·9	112·3	123·6	135·9
60	149·2	163·6	179·1	195·2	214·0	233·5	254·5	277·1	301·3	327·2
80	335·1	384·8	416·7	450·8	487·1	525·8	567·1	611·0	657·7	707·3

APPENDIX

Logarithm Tables

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						5	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	16	20	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	18	22	26	29	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	6	10	13	16	19	23	26	29
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	14	17	20	23	26
15	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	6	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	3	5	8	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	17	19	21
						2672	2695	2718	2742	2765	2	4	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1 2 3	3 4 5	6 7 8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1 2	3 4 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 6 6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	5 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	3 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 6
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 4	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 3	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 5 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0 1 1	2 2 3	3 4 4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0 1 1	2 2 3	3 4 4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0 1 1	2 2 3	3 4 4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1 1	2 2 3	3 4 4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0 1 1	2 2 3	3 4 4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1 1	2 2 3	3 4 4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0 1 1	2 2 3	3 4 4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0 1 1	2 2 3	3 4 4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0 1 1	2 2 3	3 4 4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 3 4

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	123	456	789
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0 0 1	1 1 1	2 2 2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0 0 1	1 1 1	2 2 2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1	1 1 1	2 2 2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0 0 1	1 1 1	2 2 2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0 1 1	1 1 2	2 2 2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0 1 1	1 1 2	2 2 2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0 1 1	1 1 2	2 2 2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0 1 1	1 1 2	2 2 3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0 1 1	1 1 2	2 2 3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1	1 1 2	2 2 3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0 1 1	1 2 2	2 2 3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0 1 1	1 2 2	2 2 3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0 1 1	1 2 2	2 3 3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0 1 1	1 2 2	2 3 3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0 1 1	1 2 2	2 3 3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0 1 1	1 2 2	2 3 3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0 1 1	1 2 2	2 3 3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0 1 1	1 2 2	2 3 3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0 1 1	1 2 2	3 3 3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0 1 1	1 2 2	3 3 3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0 1 1	2 2 2	3 3 3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0 1 1	2 2 2	3 3 3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0 1 1	2 2 2	3 3 4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0 1 1	2 2 2	3 3 4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0 1 1	2 2 2	3 3 4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0 1 1	2 2 3	3 3 4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0 1 1	2 2 3	3 3 4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0 1 1	2 2 3	3 4 4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0 1 1	2 2 3	3 4 4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0 1 1	2 2 3	3 4 4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0 1 1	2 2 3	3 4 4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0 1 1	2 2 3	3 4 4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0 1 1	2 2 3	3 4 4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1 1 2	2 3 3	4 4 5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2285	1 1 2	2 3 3	4 4 5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1 1 2	2 3 3	4 4 5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1 1 2	2 3 3	4 4 5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1 1 2	2 3 3	4 4 5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1 1 2	2 3 3	4 5 5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1 1 2	2 3 4	4 5 5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1 1 2	2 3 4	4 5 5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2	2 3 4	4 5 6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 1 2	3 3 4	4 5 6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 1 2	3 3 4	4 5 6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 3 4	5 6 6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 3 4	5 6 6

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	123	456	789
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1 1 2	3 4 4	5 6 7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1 2 2	3 4 5	5 6 7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1 2 2	3 4 5	5 6 7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1 2 2	3 4 5	6 6 7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1 2 2	3 4 5	6 6 7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1 2 2	3 4 5	6 7 7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1 2 3	3 4 5	6 7 8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1 2 3	3 4 5	6 7 8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1 2 3	4 4 5	6 7 8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1 2 3	4 5 5	6 7 8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1 2 3	4 5 6	6 7 8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1 2 3	4 5 6	7 8 9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1 2 3	4 5 6	7 8 9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1 2 3	4 5 6	7 8 9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1 2 3	4 5 6	7 8 9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1 2 3	4 5 6	7 8 9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1 2 3	4 5 6	7 9 10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1 2 3	4 5 7	8 9 10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1 2 3	4 6 7	8 9 10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1 2 3	5 6 7	8 9 10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1 2 4	5 6 7	8 9 11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1 2 4	5 6 7	8 10 11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1 2 4	5 6 7	9 10 11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1 3 4	5 6 8	9 10 11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1 3 4	5 6 8	9 10 12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1 3 4	5 7 8	9 10 12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1 3 4	5 7 8	9 11 12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1 3 4	5 7 8	10 11 12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1 3 4	6 7 8	10 11 13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1 3 4	6 7 9	10 11 13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1 3 4	6 7 9	10 12 13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2 3 5	6 8 9	11 12 14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2 3 5	6 8 9	11 12 14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2 3 5	6 8 9	11 13 14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2 3 5	6 8 10	11 13 15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2 3 5	7 8 10	12 13 15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2 3 5	7 8 10	12 13 15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2 3 5	7 9 10	12 14 16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2 4 5	7 9 11	12 14 16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2 4 5	7 9 11	13 14 16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2 4 6	7 9 11	13 15 17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2 4 6	8 9 11	13 15 17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2 4 6	8 10 12	14 15 17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2 4 6	8 10 12	14 16 18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2 4 6	8 10 12	14 16 18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2 4 6	8 10 12	15 17 19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2 4 6	8 11 13	15 17 19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2 4 7	9 11 13	15 17 20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2 4 7	9 11 13	16 18 20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2 5 7	9 11 14	16 18 20

NATURAL SINES

Degrees	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	Mean Differences				
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	1	2	3	4	5
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15
5	·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14
11	·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	·2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3	6	9	11	14
13	·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	·2419	2436	2452	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	·3256	3273	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	·3746	3762	3778	3795	3811	2827	3843	3859	3875	3891	3	5	8	11	14
23	·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	·5446	5461	6476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	10	12
36	·5878	5892	5906	5920	5934	5848	5962	5976	5990	6004	2	5	7	9	12
37	·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

NATURAL SINES

Degrees	0' 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' 0°.9	Mean Differences 1	2	3	4	5
45	·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	·7547	7558	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	·9205	9212	9219	9225	9232	9239	9345	9252	9259	9265	1	2	3	4	6
68	·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	·9945	9947	9949	9951	9952	8854	9956	9957	9959	9960	0	1	1	1	2
85	·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	·9998	9999	9999	9999	9999	1.000	1.000	1.000	1.000	1.000	0	0	0	0	0
90	1·000														

NATURAL COSINES

[Numbers in difference columns to be subtracted, not added.]

Degrees	0' 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' 0°.9	Mean Differences				
											1	2	3	4	5
0	1·000	1·000	1·000	1·000	1·000	1·000	9999	9999	9999	9999	0	0	0	0	0
1	·9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0	0	0	0	0
2	·9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	1	1
3	·9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	0	0	1	1	1
4	·9976	9974	9973	9972	9971	9969	9968	9966	9965	9963	0	0	1	1	1
5	·9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	1	1	1	2
6	·9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
7	·9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
8	·9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
9	·9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
10	·9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
11	·9816	9813	9810	9806	9803	9799	9796	9792	9789	9785	1	1	2	2	3
12	·9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
13	·9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
14	·9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
15	·9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
16	·9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
17	·9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	3	4
18	·9511	9505	9500	9494	9489	9483	9478	9472	9466	9461	1	2	3	4	5
19	·9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
20	·9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
21	·9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
22	·9272	9265	9259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	6
23	·9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	1	2	3	5	6
24	·9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	4	5	6
25	·9063	9056	9048	9041	9033	9026	9018	9011	9003	8996	1	3	4	5	6
26	·8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
27	·8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1	3	4	5	7
28	·8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	6	7
29	·8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
30	·8660	8652	8643	8634	8625	8616	8607	8599	8590	8581	1	3	4	6	7
31	·8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	8
32	·8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
33	·8387	8377	8368	8358	8348	8339	8329	8320	8310	8300	2	3	5	6	8
34	·8290	8281	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
35	·8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2	3	5	7	8
36	·8090	8080	8070	8059	8049	8039	8028	8018	8007	7997	2	3	5	7	9
37	·7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
38	·7880	7869	7859	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
39	·7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
40	·7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
41	·7547	7536	7524	7513	7501	7490	7478	7466	7455	7443	2	4	6	8	10
42	·7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
43	·7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
44	·7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10

NATURAL COSINES

[Numbers in difference columns to be subtracted, not added.]

Degrees	0' 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' 0°.9	Mean Differences				
											1	2	3	4	5
45	·7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2	4	6	8	10
46	·6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2	4	6	8	11
47	·6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2	4	6	9	11
48	·6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	2	4	7	9	11
49	·6561	6547	6534	6521	6508	6494	6481	6468	6455	6441	2	4	7	9	11
50	·6428	6414	6401	6388	6374	6361	6347	6334	6320	6307	2	4	7	9	11
51	·6293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2	5	7	9	11
52	·6157	6143	6129	6115	6101	6088	6074	6060	6046	6032	2	5	7	9	12
53	·6018	6004	5990	5976	5962	5948	5934	5920	5906	5892	2	5	7	9	12
54	·5878	5864	5850	5835	5821	5807	5793	5779	5764	5750	2	5	7	9	12
55	·5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2	5	7	10	12
56	·5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2	5	7	10	12
57	·5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2	5	7	10	12
58	·5299	5284	5270	5255	5240	5225	5210	5195	5180	5165	2	5	7	10	12
59	·5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3	5	8	10	13
60	·5000	4985	4970	4955	4939	4924	4900	4894	4879	4863	3	5	8	10	13
61	·4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3	5	8	10	13
62	·4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	8	10	13
63	·4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3	5	8	10	13
64	·4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3	5	8	11	13
65	·4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3	5	8	11	13
66	·4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3	5	8	11	14
67	·3907	3891	3875	3859	3843	3827	3811	3795	3778	3762	3	5	8	11	14
68	·3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3	5	8	11	14
69	·3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3	5	8	11	14
70	·3420	3404	3387	3371	3355	3338	3322	3305	3289	3272	3	5	8	11	14
71	·3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
72	·3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
73	·2924	2907	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
74	·2756	2740	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
75	·2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14
76	·2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3	6	8	11	14
77	·2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3	6	9	11	14
78	·2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3	6	9	11	14
79	·1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9	11	14
80	·1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9	12	14
81	·1564	1547	1530	1513	1495	1478	1461	1444	1426	1409	3	6	9	12	14
82	·1392	1374	1357	1340	1323	1305	1288	1271	1253	1236	3	6	9	12	14
83	·1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3	6	9	12	14
84	·1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3	6	9	12	14
85	·0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3	6	9	12	15
86	·0698	0680	0663	0645	0628	0610	0593	0576	0558	0541	3	6	9	12	15
87	·0523	0506	0488	0471	0454	0436	0419	0401	0384	0366	3	6	9	12	15
88	·0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9	12	15
89	·0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3	6	9	12	15
90	·0000														

NATURAL TANGENTS

Degrees	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	Mean Differences				
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	1	2	3	4	5
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

NATURAL TANGENTS

Degrees	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	Mean Differences				
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	1	2	3	4	5
45	1.0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5830	5941	10	20	30	40	50
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1·9626	9711	9797	9883	9970	2.0057	2.0145	2.0233	2.0323	2.0413	15	29	44	58	73
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2·9042	9208	9375	9544	9714	9887	3.0061	3.0237	3.0415	3.0595	29	58	87	116	145
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646					
78	4·7046	7453	7867	8288	8716	9152	9594	5·0045	5·0504	5·0970	Mean differences cease to be sufficiently accurate.				
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140					
80	5·6713	7297	7894	8502	9124	9758	6·0405	6·1066	6·1742	6·2432					
81	6·3138	3859	4596	5350	6122	6912	7720	8548	9395	7·0264					
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	8·0285					
83	8·1443	2636	3863	5126	6427	7769	9152	9·0579	9·2052	9·3572					
84	9·5144	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20					
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95					
86	14·30	14·67	15·06	15·46	15·89	16·35	16.83	17·34	17·89	18·46					
87	19·08	19·74	20·45	21·20	22·02	22·90	23.86	24·90	26·03	27·27					
88	28·64	30·14	31·82	33·69	35·80	38·19	40.92	44·07	47·74	52·08					
89	57·29	63·66	71·62	81·85	95·49	114·5	143.2	191·0	286·5	573·0					
90	∞														

LOGARITHMS OF SINES

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences	
											1 2 3	4 5
0	$-\infty$	$\bar{3}$·2419	$\bar{3}$·5429	7190	8439	9408	$\bar{2}$·0200	$\bar{2}$·0870	$\bar{2}$·1450	$\bar{2}$·1961		
1	$\bar{2}$·2419	2832	3210	3558	3880	4179	4459	4723	4971	5206		
2	$\bar{2}$·5428	5640	5842	6035	6220	6397	6567	6731	6889	7041		
3	$\bar{2}$·7188	7330	7468	7602	7731	7857	7979	8098	8213	8326		
4	$\bar{2}$·8436	8543	8647	8749	8849	8946	9042	9135	9226	9315	16 32 48	64 80
5	$\bar{2}$·9403	9489	9573	9655	9736	9816	9894	9970	$\bar{1}$.0046	$\bar{1}$.0120	13 26 39	52 65
6	$\bar{1}$·0192	0264	0334	0403	0472	0539	0605	0670	0734	0797	11 22 33	44 55
7	$\bar{1}$·0859	0900	0981	1040	1099	1157	1214	1271	1326	1381	10 19 29	38 48
8	$\bar{1}$·1436	1489	1542	1594	1646	1697	1747	1797	1847	1895	8 17 25	34 42
9	$\bar{1}$·1943	1991	2038	2085	2131	2176	2221	2266	2310	2353	8 15 23	30 38
10	$\bar{1}$·2397	2439	2482	2524	2565	2606	2647	2687	2727	2767	7 14 20	27 34
11	$\bar{1}$·2806	2845	2883	2921	2959	2997	3034	3070	3107	3143	6 12 19	25 31
12	$\bar{1}$·3179	3214	3250	3284	3319	3353	3387	3421	3455	3488	6 11 17	23 28
13	$\bar{1}$·3521	3554	3585	3618	3650	3682	3713	3745	3775	3806	5 11 16	21 26
14	$\bar{1}$·3837	3867	3897	3927	3957	3986	4015	4044	4073	4102	5 10 15	20 24
15	$\bar{1}$·4130	4158	4186	4214	4242	4269	4296	4323	4350	4377	5 9 14	18 23
16	$\bar{1}$·4403	4430	4456	4482	4508	4533	4559	4584	4609	4634	4 9 13	17 21
17	$\bar{1}$·4659	4684	4709	4733	4757	4781	4805	4829	4853	4876	4 8 12	16 20
18	$\bar{1}$·4900	4923	4946	4969	4992	5015	5037	5060	5082	5104	4 8 11	15 19
19	$\bar{1}$·5126	5148	5170	5192	5213	5235	5256	5278	5299	5320	4 7 11	14 18
20	$\bar{1}$·5341	5361	5382	5402	5423	5443	5463	5484	5504	5523	3 7 10	14 17
21	$\bar{1}$·5543	5563	5583	5602	5621	5641	5660	5679	5698	5717	3 6 10	13 16
22	$\bar{1}$·5736	5754	5773	5792	5810	5828	5847	5865	5883	5901	3 6 9	12 15
23	$\bar{1}$·5919	5937	5954	5972	5990	6007	6024	6042	6059	6076	3 6 9	12 15
24	$\bar{1}$·6093	6110	6127	6144	6161	6177	6194	6210	6227	6243	3 6 8	11 14
25	$\bar{1}$·6259	6276	6292	6308	6324	6340	6356	6371	6387	6403	3 5 8	11 13
26	$\bar{1}$·6418	6434	6449	6465	6480	6495	6510	6526	6541	6556	3 5 8	10 13
27	$\bar{1}$·6570	6585	6600	6615	6629	6644	6659	6673	6687	6702	2 5 7	10 12
28	$\bar{1}$·6716	6730	6744	6759	6773	6787	6801	6814	6828	6842	2 5 7	9 12
29	$\bar{1}$·6856	6869	6883	6896	6910	6923	6937	6950	6863	6977	2 4 7	9 11
30	$\bar{1}$·6990	7003	7016	7029	7042	7055	7068	7080	7093	7106	2 4 6	9 11
31	$\bar{1}$·7118	7131	7144	7156	7168	7181	7193	7205	7218	7230	2 4 6	8 10
32	$\bar{1}$·7242	7254	7266	7278	7290	7302	7314	7326	7338	7349	2 4 6	8 10
33	$\bar{1}$·7361	7373	7384	7396	7407	7419	7430	7442	7453	7464	2 4 6	8 10
34	$\bar{1}$·7476	7487	7498	7509	7520	7531	7542	7553	7564	7575	2 4 6	7 9
35	$\bar{1}$·7586	7597	7607	7618	7629	7640	7650	7661	7671	7682	2 4 5	7 9
36	$\bar{1}$·7692	7703	7713	7723	7734	7744	7754	7764	7774	7785	2 3 5	7 9
37	$\bar{1}$·7795	7805	7815	7825	7835	7844	7854	7864	7874	7884	2 3 5	7 8
38	$\bar{1}$·7893	7903	7913	7922	7932	7941	7951	7960	7970	7979	2 3 5	6 8
39	$\bar{1}$·7989	7998	8007	8017	8026	8035	8044	8053	8063	8072	2 3 5	6 8
40	$\bar{1}$·8081	8090	8099	8108	8117	8125	8134	8143	8152	8161	1 3 4	6 7
41	$\bar{1}$·8169	8178	8187	8195	8204	8213	8221	8230	8238	8247	1 3 4	6 7
42	$\bar{1}$·8255	8264	8272	8280	8289	8297	8305	8313	8322	8330	1 3 4	6 7
43	$\bar{1}$·8338	8346	8354	8362	8370	8378	8386	8394	8402	8410	1 3 4	5 7
44	$\bar{1}$·8418	8426	8433	8441	8449	8457	8464	8472	8480	8487	1 3 4	5 6

LOGARITHMS OF SINES

Degrees	0' 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' 0°.9	Mean Differences 1 2 3	4 5
45	$\bar{1}$·8495	8502	8510	8517	8525	8532	8540	8547	8555	8562	1 2 4	5 6
46	$\bar{1}$·8569	8577	8584	8591	8598	8606	8613	8620	8627	8634	1 2 4	5 6
47	$\bar{1}$·8641	8648	8655	8662	8669	8676	8683	8690	8697	8704	1 2 3	5 6
48	$\bar{1}$·8711	8718	8724	8731	8738	8745	8751	8758	8765	8771	1 2 3	4 6
49	$\bar{1}$·8778	8784	8791	8797	8804	8810	8817	8823	8830	8836	1 2 3	4 5
50	$\bar{1}$·8843	8849	8855	8862	8868	8874	8880	8887	8893	8899	1 2 3	4 5
51	$\bar{1}$·8905	8911	8917	8923	8929	8935	8941	8947	8953	8959	1 2 3	4 5
52	$\bar{1}$·8965	8971	8977	8983	8989	8995	9000	9006	9012	9018	1 2 3	4 5
53	$\bar{1}$·9023	9029	9035	9041	9046	9052	9057	9063	9069	9074	1 2 3	4 5
54	$\bar{1}$·9080	9085	9091	9096	9101	9107	9112	9118	9123	9128	1 2 3	4 5
55	$\bar{1}$·9134	9139	9144	9149	9155	9160	9165	9170	9175	9181	1 2 3	3 4
56	$\bar{1}$·9186	9191	9196	9201	9206	9211	9216	9221	9226	9231	1 2 3	3 4
57	$\bar{1}$·9236	9241	9246	9251	9255	9260	9265	9270	9275	9279	1 2 2	3 4
58	$\bar{1}$·9284	9289	9294	9298	9303	9308	9312	9317	9322	9326	1 2 2	3 4
59	$\bar{1}$·9331	9335	9340	9344	9349	9353	9358	9362	9367	9371	1 1 2	3 4
60	$\bar{1}$·9375	9380	9384	9388	9393	9397	9401	9406	9410	9414	1 1 2	3 4
61	$\bar{1}$·9418	9422	9427	9431	9435	9439	9443	9447	9451	9455	1 1 2	3 3
62	$\bar{1}$·9459	9463	9467	9471	9475	9479	9483	9487	9491	9495	1 1 2	3 3
63	$\bar{1}$·9499	9503	9506	9510	9514	9518	9522	9525	9529	9533	1 1 2	3 3
64	$\bar{1}$·9537	9540	9544	9548	9551	9555	9558	9562	9566	9569	1 1 2	2 3
65	$\bar{1}$·9573	9576	9580	9583	9587	9590	9594	9597	9601	9604	1 1 2	2 3
66	$\bar{1}$·9607	9610	9614	9617	9621	9624	9627	9631	9634	9637	1 1 2	2 3
67	$\bar{1}$·9640	9643	9647	9650	9653	9656	9659	9662	9666	9669	1 1 2	2 3
68	$\bar{1}$·9672	9674	9678	9681	9684	9687	9690	9693	9696	9699	0 1 1	2 2
69	$\bar{1}$·9702	9704	9707	9710	9713	9716	9719	9722	9724	9727	0 1 1	2 2
70	$\bar{1}$·9730	9733	9735	9738	9741	9743	9746	9749	9751	9754	0 1 1	2 2
71	$\bar{1}$·9757	9759	9762	9764	9767	9770	9772	9775	9777	9780	0 1 1	2 2
72	$\bar{1}$·9782	9785	9787	9789	9792	9794	9797	9799	9801	9804	0 1 1	2 2
73	$\bar{1}$·9806	9808	9811	9813	9815	9817	9820	9822	9824	9826	0 1 1	2 2
74	$\bar{1}$·9828	9831	9833	9835	9837	9839	9841	9843	9845	9847	0 1 1	1 2
75	$\bar{1}$·9849	9851	9853	9855	9857	9859	9861	9863	9865	9867	0 1 1	1 2
76	$\bar{1}$·9869	9871	9873	9875	9876	9878	9880	9882	9884	9885	0 1 1	1 2
77	$\bar{1}$·9887	9889	9891	9892	9894	9896	9897	9899	9901	9902	0 1 1	1 1
78	$\bar{1}$·9904	9906	9907	9909	9910	9912	9913	9915	9916	9918	0 1 1	1 1
79	$\bar{1}$·9919	9921	9922	9924	9925	9927	9928	9929	9931	9932	0 0 1	1 1
80	$\bar{1}$·9934	9935	9936	9937	9939	9940	9941	9943	9944	9945	0 0 1	1 1
81	$\bar{1}$·9946	9947	9949	9950	9951	9952	9953	9954	9955	9956	0 0 1	1 1
82	$\bar{1}$·9958	9959	9960	9961	9962	9963	9964	9965	9966	9967	0 0 1	1 1
83	$\bar{1}$·9968	9968	9969	9970	9971	9972	9973	9974	9975	9975	0 0 0	1 1
84	$\bar{1}$·9976	9977	9978	9978	9979	9980	9981	9981	9982	9983	0 0 0	0 1
85	$\bar{1}$·9983	9984	9985	9985	9986	9987	9987	9988	9988	9989	0 0 0	0 0
86	$\bar{1}$·9989	9990	9990	9991	9991	9992	9992	9993	9993	9994	0 0 0	0 0
87	$\bar{1}$·9994	9994	9995	9995	9996	9996	9996	9996	9997	9997	0 0 0	0 0
88	$\bar{1}$·9997	9998	9998	9998	9998	9999	9999	9999	9999	9999	0 0 0	0 0
89	$\bar{1}$·9999	9999	0.0000	0000	0000	0000	0000	0000	0000	0000		
90	0·0000											

LOGARITHMS OF COSINES

[Numbers in difference columns to be subtracted, not added.]

Degrees	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	Mean Differences	
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	1 2 3	4 5
0	0.0000	0000	0000	0000	0000	0000	0000	0000	0000	$\bar{1}$.9999	0 0 0	0 0
1	$\bar{1}$.9999	9999	9999	9999	9999	9999	9998	9998	9998	9998	0 0 0	0 0
2	$\bar{1}$.9997	9997	9997	9996	9996	9996	9996	9995	9995	9994	0 0 0	0 0
3	$\bar{1}$.9994	9994	9993	9993	9992	9992	9991	9991	9990	9990	0 0 0	0 0
4	$\bar{1}$.9989	9989	9988	9988	9987	9987	9986	9985	9985	9984	0 0 0	0 0
5	$\bar{1}$.9983	9983	9982	9981	9981	9980	9979	9978	9978	9977	0 0 0	0 1
6	$\bar{1}$.9976	9975	9975	9974	9973	9972	9971	9970	9969	9968	0 0 0	1 1
7	$\bar{1}$.9968	9967	9966	9965	9964	9963	9962	9961	9960	9959	0 0 1	1 1
8	$\bar{1}$.9958	9956	9955	9954	9953	9952	9951	9950	9949	9947	0 0 1	1 1
9	$\bar{1}$.9946	9945	9944	9943	9941	9940	9939	9937	9936	9935	0 0 1	1 1
10	$\bar{1}$.9934	9932	9931	9929	9928	9927	9925	9924	9922	9921	0 0 1	1 1
11	$\bar{1}$.9919	9918	9916	9915	9913	9912	9910	9909	9907	9906	0 1 1	1 1
12	$\bar{1}$.9904	9902	9901	9899	9897	9896	9894	9892	9891	9889	0 1 1	1 1
13	$\bar{1}$.9887	9885	9884	9882	9880	9878	9876	9875	9873	9871	0 1 1	1 2
14	$\bar{1}$.9869	9867	9865	9863	9861	9859	9857	9855	9853	9851	0 1 1	1 2
15	$\bar{1}$.9849	9847	9845	9843	9841	9839	9837	9835	9833	9831	0 1 1	1 2
16	$\bar{1}$.9828	9826	9824	9822	9820	9817	9815	9813	9811	9808	0 1 1	2 2
17	$\bar{1}$.9806	9804	9801	9799	9797	9794	9792	9789	9787	9785	0 1 1	2 2
18	$\bar{1}$.9782	9780	9777	9775	9772	9770	9767	9764	9762	9759	0 1 1	2 2
19	$\bar{1}$.9757	9754	9751	9749	9746	9743	9741	9738	9735	9733	0 1 1	2 2
20	$\bar{1}$.9730	9727	9724	9722	9719	9716	9713	9710	9707	9704	0 1 1	2 2
21	$\bar{1}$.9702	9699	9696	9693	9690	9687	9684	9681	9678	9675	0 1 1	2 2
22	$\bar{1}$.9672	9669	9666	9662	9659	9656	9653	9650	9647	9643	1 1 2	2 3
23	$\bar{1}$.9640	9637	9634	9631	9627	9624	9621	9617	9614	9611	1 1 2	2 3
24	$\bar{1}$.9607	9604	9601	9597	9594	9590	9587	9583	9580	9576	1 1 2	2 3
25	$\bar{1}$.9573	9569	9566	9562	9558	9555	9551	9548	9544	9540	1 1 2	2 3
26	$\bar{1}$.9537	9533	9529	9525	9522	9518	9514	9510	9506	9503	1 1 2	3 3
27	$\bar{1}$.9499	9495	9491	9487	9483	9479	9475	9471	9467	9463	1 1 2	3 3
28	$\bar{1}$.9459	9455	9451	9447	9443	9439	9435	9431	9427	9422	1 1 2	3 3
29	$\bar{1}$.9418	9414	9410	9406	9401	9397	9393	9388	9384	9380	1 1 2	3 4
30	$\bar{1}$.9375	9371	9367	9362	9358	9353	9349	9344	9340	9335	1 1 2	3 4
31	$\bar{1}$.9331	9326	9322	9317	9312	9308	9303	9298	9294	9289	1 2 2	3 4
32	$\bar{1}$.9284	9279	9275	9270	9265	9260	9255	9251	9246	9241	1 2 2	3 4
33	$\bar{1}$.9236	9231	9226	9221	9216	9211	9206	9201	9196	9191	1 2 3	3 4
34	$\bar{1}$.9186	9181	9175	9170	9165	9160	9155	9149	9144	9139	1 2 3	3 4
35	$\bar{1}$.9134	9128	9123	9118	9112	9107	9101	9096	9091	9085	1 2 3	4 5
36	$\bar{1}$.9080	9074	9069	9063	9057	9052	9046	9041	9035	9029	1 2 3	4 5
37	$\bar{1}$.9023	9018	9012	9006	9000	8995	8989	8983	8977	8971	1 2 3	4 5
38	$\bar{1}$.8965	8959	8953	8947	8941	8935	8929	8923	8917	8911	1 2 3	4 5
39	$\bar{1}$.8905	8899	8893	8887	8880	8874	8868	8862	8855	8840	1 2 3	4 5
40	$\bar{1}$.8843	8836	8830	8823	8817	8810	8804	8797	8791	8784	1 2 3	4 5
41	$\bar{1}$.8778	8771	8765	8758	8751	8745	8738	8731	8724	8718	1 2 3	5 6
42	$\bar{1}$.8711	8704	8697	8690	8683	8676	8669	8662	8655	8648	1 2 3	5 6
43	$\bar{1}$.8641	8634	8627	8620	8613	8606	8598	8591	8584	8577	1 2 4	5 6
44	$\bar{1}$.8569	8562	8555	8547	8540	8532	8525	8517	8510	8502	1 2 4	5 6

LOGARITHMS OF COSINES

[Numbers in difference columns to be subtracted, not added].

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences				
											1	2	3	4	5
45	$\bar{1}$.8495	8487	8480	8472	8464	8457	8449	8441	8433	8426	1	3	4	5	6
46	$\bar{1}$.8418	8410	8402	8394	8386	8378	8370	8362	8354	8346	1	3	4	5	7
47	$\bar{1}$.8338	8330	8322	8313	8305	8297	8289	8280	8272	8264	1	3	4	6	7
48	$\bar{1}$.8255	8247	8238	8230	8221	8213	8204	8195	8187	8178	1	3	4	6	7
49	$\bar{1}$.8169	8161	8152	8143	8134	8125	8117	8108	8099	8090	1	3	4	6	7
50	$\bar{1}$.8081	8072	8063	8053	8044	8035	8026	8017	8007	7998	2	3	5	6	8
51	$\bar{1}$.7989	7979	7970	7960	7951	7941	7932	7922	7913	7903	2	3	5	6	8
52	$\bar{1}$.7893	7884	7874	7864	7854	7844	7835	7825	7815	7805	2	3	5	7	8
53	$\bar{1}$.7795	7785	7774	7764	7754	7744	7734	7723	7713	7703	2	3	5	7	9
54	$\bar{1}$.7692	7682	7671	7661	7650	7640	7629	7618	7607	7597	2	4	5	7	9
55	$\bar{1}$.7586	7575	7564	7553	7542	7531	7520	7509	7498	7487	2	4	6	7	9
56	$\bar{1}$.7476	7464	7453	7442	7430	7419	7407	7396	7384	7373	2	4	6	8	10
57	$\bar{1}$.7361	7349	7338	7326	7314	7302	7290	7278	7266	7254	2	4	6	8	10
58	$\bar{1}$.7242	7230	7218	7205	7193	7181	7168	7156	7144	7131	2	4	6	8	10
59	$\bar{1}$.7118	7106	7093	7080	7068	7055	7042	7029	7016	7003	2	4	6	9	11
60	$\bar{1}$.6990	6977	6963	6950	6937	6923	6910	6896	6883	6869	2	4	7	9	11
61	$\bar{1}$.6856	6842	6828	6814	6801	6787	6773	6759	6744	6730	2	5	7	9	12
62	$\bar{1}$.6716	6702	6687	6673	6659	6644	6629	6615	6600	6585	2	5	7	10	12
63	$\bar{1}$.6570	6556	6541	6526	6510	6495	6480	6465	6449	6434	3	5	8	10	13
64	$\bar{1}$.6418	6403	6387	6371	6356	6340	6324	6308	6292	6276	3	5	8	11	13
65	$\bar{1}$.6259	6243	6227	6210	6194	6177	6161	6144	6127	6110	3	6	8	11	14
66	$\bar{1}$.6093	6076	6059	6042	6024	6007	5990	5972	5954	5937	3	6	9	12	15
67	$\bar{1}$.5919	5901	5883	5865	5847	5828	5810	5792	5773	5754	3	6	9	12	15
68	$\bar{1}$.5736	5717	5698	5679	5660	5641	5621	5602	5583	5563	3	6	10	13	16
69	$\bar{1}$.5543	5523	5504	5484	5463	5443	5423	5402	5382	5361	3	7	10	14	17
70	$\bar{1}$.5341	5320	5299	5278	5256	5235	5213	5192	5170	5148	4	7	11	14	18
71	$\bar{1}$.5126	5104	5082	5060	5037	5015	4992	4969	4946	4923	4	8	11	15	19
72	$\bar{1}$.4900	4876	4853	4829	4805	4781	4757	4733	4709	4684	4	8	12	16	20
73	$\bar{1}$.4659	4634	4609	4584	4559	4533	4508	4482	4456	4430	4	9	13	17	21
74	$\bar{1}$.4403	4377	4350	4323	4296	4269	4242	4214	4186	4158	5	9	14	18	23
75	$\bar{1}$.4130	4102	4073	4044	4015	3986	3957	3927	3897	3867	5	10	15	20	24
76	$\bar{1}$.3837	3806	3775	3745	3713	3682	3650	3618	3586	3554	5	11	16	21	26
77	$\bar{1}$.3521	3488	3455	3421	3387	3353	3319	3284	3250	3214	6	11	17	23	28
78	$\bar{1}$.3179	3143	3107	3070	3034	2997	2959	2921	2883	2845	6	12	19	25	31
79	$\bar{1}$.2806	2767	2727	2687	2647	2606	2565	2524	2482	2439	7	14	20	27	34
80	$\bar{1}$.2397	2353	2310	2266	2221	2176	2131	2085	2038	1991	8	15	23	30	38
81	$\bar{1}$.1943	1895	1847	1797	1747	1697	1646	1594	1542	1489	8	17	25	34	42
82	$\bar{1}$.1436	1381	1326	1271	1214	1157	1099	1040	0981	0920	10	19	29	38	48
83	$\bar{1}$.0859	0797	0734	0670	0605	0539	0472	0403	0334	0264	11	22	33	44	55
84	$\bar{1}$.0192	0120	0046	$\bar{2}$.9970	$\bar{2}$.9894	$\bar{2}$.9816	$\bar{2}$.9736	$\bar{2}$.9655	$\bar{2}$.9573	$\bar{2}$.9489	13	26	39	52	65
85	$\bar{2}$.9403	9315	9226	9135	9042	8946	8849	8749	8647	8543	16	32	48	64	80
86	$\bar{2}$.8436	8326	8213	8098	7979	7857	7731	7602	7468	7330					
87	$\bar{2}$.7188	7041	6889	6731	6567	6397	6220	6035	5842	5640					
88	$\bar{2}$.5428	5206	4971	4723	4459	4179	3880	3558	3210	2832					
89	$\bar{2}$.2419	1961	1450	0870	0200	$\bar{3}$.9408	$\bar{3}$.8439	$\bar{3}$.7190	$\bar{3}$.5429	$\bar{3}$.2419					
90	∞														

LOGARITHMS OF TANGENTS

Degrees	0' 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' .0°.9	Mean Differences				
											1	2	3	4	5
0	−∞	$\bar{3}.2419$	$\bar{3}.5429$	$\bar{3}.7190$	$\bar{3}.8439$	$\bar{3}.9409$	$\bar{2}.0200$	$\bar{2}.0870$	$\bar{2}.1450$	$\bar{2}.1962$					
1	$\bar{2}.2419$	2833	3211	3559	3881	4181	4461	4725	4973	5208					
2	$\bar{2}.5431$	5643	5845	6038	6223	6401	6571	6736	6894	7046					
3	$\bar{2}.7194$	7337	7475	7609	7739	7865	7988	8107	8223	8336					
4	$\bar{2}.8446$	8554	8659	8762	8862	8960	9056	9150	9241	9331	16	32	48	64	81
5	$\bar{2}.9420$	9506	9591	9674	9756	9836	9915	9992	$\bar{1}.0068$	$\bar{1}.0143$	13	26	40	53	66
6	$\bar{1}.0216$	0289	0360	0430	0499	0567	0633	0699	0764	0828	11	22	34	45	56
7	$\bar{1}.0891$	0954	1015	1076	1135	1194	1252	1310	1367	1423	10	20	29	39	49
8	$\bar{1}.1478$	1533	1587	1640	1693	1745	1797	1848	1898	1948	9	17	26	35	43
9	$\bar{1}.1997$	2046	2094	2142	2189	2236	2282	2328	2374	2419	8	16	23	31	39
10	$\bar{1}.2463$	2507	2551	2594	2637	2680	2722	2764	2805	2846	7	14	21	28	35
11	$\bar{1}.2887$	2927	2967	3006	3046	3085	3123	3162	3200	3237	6	13	19	26	32
12	$\bar{1}.3275$	3312	3349	3385	3422	3458	3493	3529	3564	3599	6	12	18	24	30
13	$\bar{1}.3634$	3668	3702	3736	3770	3804	3837	3870	3903	3935	6	11	17	22	28
14	$\bar{1}.3968$	4000	4032	4064	4095	4127	4158	4189	4220	4250	5	10	16	21	26
15	$\bar{1}.4281$	4311	4341	4371	4400	4430	4459	4488	4517	4646	5	10	15	20	25
16	$\bar{1}.4575$	4603	4632	4660	4688	4716	4744	4771	4799	4826	5	9	14	19	23
17	$\bar{1}.4853$	4880	4907	4934	4961	4987	5014	5040	5066	5092	4	9	13	18	22
18	$\bar{1}.5118$	5143	5169	5195	5220	5245	5270	5295	5320	5345	4	8	13	17	21
19	$\bar{1}.5370$	5394	5419	5443	5467	5491	5516	5539	5563	5587	4	8	12	16	20
20	$\bar{1}.5611$	5634	5658	5681	5704	5727	5750	5773	5796	5819	4	8	12	15	19
21	$\bar{1}.5842$	5864	5887	5909	5932	5954	5976	5998	6020	6042	4	7	11	15	19
22	$\bar{1}.6064$	6086	6108	6129	6151	6172	6194	6215	6236	6257	4	7	11	14	18
23	$\bar{1}.6279$	6300	6321	6341	6362	6383	6404	6424	6445	6465	3	7	10	14	17
24	$\bar{1}.6486$	6506	6527	6547	6567	6587	6607	6627	6647	6667	3	7	10	13	17
25	$\bar{1}.6687$	6706	6726	6746	6765	6785	6804	6824	6843	6863	3	7	10	13	16
26	$\bar{1}.6882$	6901	6920	6939	6958	6977	6996	7015	7034	7053	3	6	9	13	16
27	$\bar{1}.7072$	7090	7109	7128	7146	7165	7183	7202	7220	7238	3	6	9	12	15
28	$\bar{1}.7257$	7275	7293	7311	7330	7348	7366	7384	7402	7420	3	6	9	12	15
29	$\bar{1}.7438$	7455	7473	7491	7509	7526	7544	7562	7579	7597	3	6	9	12	15
30	$\bar{1}.7614$	7632	7649	7667	7684	7701	7719	7736	7753	7771	3	6	9	12	14
31	$\bar{1}.7788$	7805	7822	7839	7856	7873	7890	7907	7924	7941	3	6	9	11	14
32	$\bar{1}.7958$	7975	7992	8008	8025	8042	8059	8075	8092	8109	3	6	8	11	14
33	$\bar{1}.8125$	8142	8158	8175	8191	8208	8224	8241	8257	8274	3	5	8	11	14
34	$\bar{1}.8290$	8306	8323	8339	8355	8371	8388	8404	8420	8436	3	5	8	11	14
35	$\bar{1}.8452$	8468	8484	8501	8517	8533	8549	8565	8581	8597	3	5	8	11	13
36	$\bar{1}.8613$	8629	8644	8660	8676	8692	8708	8724	8740	8755	3	5	8	11	13
37	$\bar{1}.8771$	8787	8803	8818	8834	8850	8865	8881	8897	8912	3	5	8	10	13
38	$\bar{1}.8928$	8944	8959	8975	8990	9006	9022	9037	9053	9068	3	5	8	10	13
39	$\bar{1}.9084$	9099	9115	9130	9146	9161	9176	9192	9207	9223	3	5	8	10	13
40	$\bar{1}.9238$	9254	9269	9284	9300	9315	9330	9346	9361	9376	3	5	8	10	13
41	$\bar{1}.9392$	9407	9422	9438	9453	9468	9483	9499	9514	9529	3	5	8	10	13
42	$\bar{1}.9544$	9560	9575	9590	9605	9621	9636	9651	9666	9681	3	5	8	10	13
43	$\bar{1}.9697$	9712	9727	9742	9757	9772	9788	9803	9818	9833	3	5	8	10	13
44	$\bar{1}.9848$	9864	9879	9894	9909	9924	9939	9955	9970	9985	3	5	8	10	13

LOGARITHMS OF TANGENTS

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences				
											1	2	3	4	5
45	·0000	0015	0030	0045	0061	0076	0091	0106	0121	0136	3	5	8	10	13
46	·0152	0167	0182	0197	0212	0228	0243	0258	0273	0288	3	5	8	10	13
47	·0303	0319	0334	0349	0364	0379	0395	0410	0425	0440	3	5	8	10	13
48	·0456	0471	0486	0501	0517	0532	0547	0562	0578	0593	3	5	8	10	13
49	·0608	0624	0639	0654	0670	0685	0700	0716	0731	0746	3	5	8	10	13
50	·0762	0777	0793	0808	0824	0839	0854	0870	0885	0901	3	5	8	10	13
51	·0916	0932	0947	0963	0978	0994	1010	1025	1041	1056	3	5	8	10	13
52	·1072	1088	1103	1119	1135	1150	1166	1182	1197	1213	3	5	8	10	13
53	·1229	1245	1260	1276	1292	1308	1324	1340	1356	1371	3	5	8	11	13
54	·1387	1403	1419	1435	1451	1467	1483	1499	1516	1532	3	5	8	11	13
55	·1548	1564	1580	1596	1612	1629	1645	1661	1677	1694	3	5	8	11	14
56	·1710	1726	1743	1759	1776	1792	1809	1825	1842	1858	3	5	8	11	14
57	·1875	1891	1908	1925	1941	1958	1975	1992	2008	2025	3	6	8	11	14
58	·2042	2059	2076	2093	2110	2127	2144	2161	2178	2195	3	6	9	11	14
59	·2212	2229	2247	2264	2281	2299	2316	2333	2351	2368	3	6	9	12	14
60	·2386	2403	2421	2438	2456	2474	2491	2509	2527	2545	3	6	9	12	15
61	·2562	2580	2598	2616	2634	2652	2670	2689	2707	2725	3	6	9	12	15
62	·2743	2762	2780	2798	2817	2835	2854	2872	2891	2910	3	6	9	12	15
63	·2928	2947	2966	2985	3004	3023	3042	3061	3080	3099	3	6	9	13	16
64	·3118	3137	3157	3176	3196	3215	3235	3254	3274	3294	3	6	10	13	16
65	·3313	3333	3353	3373	3393	3413	3433	3453	3473	3494	3	7	10	13	17
66	·3514	3535	3555	3576	3596	3617	3638	3659	3679	3700	3	7	10	14	17
67	·3721	3743	3764	3785	3806	3828	3849	3871	3892	3914	4	7	11	14	18
68	·3936	3958	3980	4002	4024	4046	4068	4091	4113	4136	4	7	11	15	19
69	·4158	4181	4204	4227	4250	4273	4296	4319	4342	4366	4	8	12	15	19
70	·4389	4413	4437	4461	4484	4509	4533	4557	4581	4606	4	8	12	16	20
71	·4630	4655	4680	4705	4730	4755	4780	4805	4831	4857	4	8	13	17	21
72	·4882	4908	4934	4960	4986	5013	5039	5066	5093	5120	4	9	13	18	22
73	·5147	5174	5201	5229	5256	5284	5312	5340	5368	5397	5	9	14	19	23
74	·5425	5454	5483	5512	5541	5570	5600	5629	5659	5689	5	10	15	20	25
75	·5719	5750	5780	5811	5842	5873	5905	5936	5968	6000	5	10	16	21	26
76	·6032	6065	6097	6130	6163	6196	6230	6264	6298	6332	6	11	17	22	28
77	·6366	6401	6436	6471	6507	6542	6578	6615	6651	6688	6	12	18	24	30
78	·6725	6763	6800	6838	6877	6915	6954	6904	7033	7073	6	13	19	26	32
79	·7113	7154	7195	7236	7278	7320	7363	7406	7449	7493	7	14	21	28	35
80	·7537	7581	7626	7672	7718	7764	7811	7858	7906	7954	8	16	23	31	39
81	·8003	8052	8102	8152	8203	8255	8307	8360	8413	8467	9	17	26	35	43
82	·8522	8577	8633	8690	8748	8806	8865	8924	8985	9046	10	20	29	39	49
83	·9109	9172	9236	9301	9367	9433	9501	9570	9640	9711	11	22	34	45	56
84	·9784	9857	9932	1·0008	1·0085	1·0164	1·0244	1·0326	1·0409	1·0494	13	26	40	53	66
85	1·0580	0669	0759	0850	0944	1040	1138	1238	1341	1446	16	32	48	64	81
86	1·1554	1664	1777	1893	2012	2135	2261	2391	2525	2663					
87	1·2806	2954	3106	3264	3429	3599	3777	3962	4155	4357					
88	1·4569	4792	5027	5275	5539	5819	6119	6441	6789	7167					
89	1·7581	8038	8550	9130	9800	2·0591	2·1561	2·2810	2·4571	2·7581					